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Homeworks on international risk sharing

1. Consider a two country world and assume that the period utility function of the representative agent is given by $U(c, l)$ where l is employment population ratio and c an aggregate of per capita consumption of tradable c_T and non tradable goods and services c_N.

$$c = c_T^{\alpha} c_N^{1-\alpha}$$

Construct series for real consumption of tradable and non tradable for US,UK Canada and Japan. (there is no unique way of doing this so you should search for different data sources, one good place to start is to look up OECD databases) and for employment population ratio. Calibrate a country specific α parameter using your consumption data. Consider four types of utility functions

$$\begin{aligned}
U(c, l) &= \frac{(c_T + c_N)^{1-\sigma}}{1-\sigma} \\
U(c, l) &= \frac{c^{1-\sigma}}{1-\sigma} \\
U(c, l) &= \frac{\left[c^{\mu}(1-l)^{1-\mu}\right]^{1-\sigma}}{1-\sigma} \\
U(c, l) &= \frac{\left[c - \delta \frac{l^{\gamma}}{\gamma}\right]^{1-\sigma}}{1-\sigma}
\end{aligned}$$

In a world with a single traded good which can be costlessly traded perfect risk sharing between the two countries implies that the ratio of marginal utilities from traded goods the two countries should be constant, or alternatively the growth in marginal utilities across countries should be perfectly correlated . For all the 6 country pairs you have data for assess how far you are from perfect risk sharing using the correlation of marginal utility growth. In order to compute marginal utility you have to take a stand on parameters value.Set $\sigma = 2$ and $\gamma = 2$. Set μ and δ such that in a steady state equilibrium in which consumption is 1 the employment popiulation ratio in the model is 1/2.Assess the role of non tradable and leisure in measuring international risk sharing.

2. Consider the following version of the Backus-Smith economy with limited enforcement. Period utility function is given by

$$\begin{aligned} u(c) &= \frac{[c]^{1-\sigma}}{1-\sigma} \\ c &= G(a,b) = (\omega a^{\frac{\gamma-1}{\gamma}} + (1-\omega) b^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}} \end{aligned}$$

The equal weigths planning problem is given by

$$\max E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + u(c_t^*) \right]$$

s.t.

$$\begin{aligned} A_t + A_t^* &= a_t + a_t^* \\ G(a_t, b_t) &= c_t \\ G(a_t^*, b_t^*) &= c_t^* \end{aligned}$$

$$u(c_t) + E \sum_{r=j+1}^{\infty} \beta^{r-t} u(c_r) \geq V_t^A \quad (1)$$

$$u(c_t^*) + E \sum_{r=t+1}^{\infty} \beta^{r-t} u(c_r^*) \geq V_t^{A^*} \quad (2)$$

where

$$\begin{aligned} V_t^A &= u(G(A_t, b_t)) + \beta E V_{t+1}^A \\ V_t^{A^*} &= u(G(A_t^*, b_t^*)) + \beta E V_{t+1}^{A*} \end{aligned}$$

where A, A^* and b, b^* are the exogenous stochastic endowments of the (single) tradable good and the (country specific) non tradable good. Assume that A, A^* and b, b^* follow a Markov chain with 16 possible states (each endowment in each country can either be high or low). Assume that the Markov chain is symmetric and uncorrelated across countries, across goods and across states and set the states and transition matrix so that volatility of each endowment is 2% and that the average duration of a state is 5 periods. Set $\sigma = 2, \gamma = 1.5$ and $\beta = 0.96$.Numericallly find the solution of the planning problem, in particular solve for decision rules $a(A, A^*, b, b^*, z)$ and $a^*(A, A^*, b, b^*, z)$ where z is the auxiliary state variable described in class. Simulate the model and report volatility of the real exchange rate and its correlation with relative consumption. Compare your findings with the allocation without enforcement constraints (complete markets).