



Economic Growth and Development (8311)

Fall 2006, Mini 2

Problem set 3

Due Monday, January 29, 2007 by email.

Consider a discrete time economy.. At each date t a new cohort of a continuum of measure μ agents is born. All agents in each cohort live for 20 periods and then die for sure. Let $a_{i,j,t}$, $z_{i,j,t}$, $c_{i,j,t}$ and $l_{i,j,t}$ denote assets, labor productivity, consumption and time devoted to work of agent i , of age j at time t . Each agent is endowed with 1 unit of time each period and is born with 0 assets. Her preferences at birth are given by

$$E_t \sum_{j=0}^{30} \beta^j U(c_{i,j,t+j}, l_{i,j,t+j})$$

where

$$U(c, l) = \log(c) - \frac{1}{\xi} l^\xi$$

her budget constraint is given by

$$\begin{aligned} a_{i,j,t}(1+r) + w l_{i,j,t} z_{i,j,t} &\geq c_{i,j,t} + a_{i,j+1,t+1} \\ a_{i,j+1,t+1} &\geq 0 \end{aligned}$$

where w and r are (constant) wage and interest rate. Assume that idiosyncratic labor productivity $\log(z_{i,j,t})$ follows a Markov chain with state vector and transition

probability matrix given by

$$\begin{bmatrix} -h & 0 & h \\ p & \frac{1-p}{2} & \frac{1-p}{2} \\ \frac{1-p}{2} & p & \frac{1-p}{2} \\ \frac{1-p}{2} & \frac{1-p}{2} & p \end{bmatrix}$$

Assume that that when agents are born they draw their initial value of z from the stationary distribution of z . There are competitive firms, which produce a single good which is used for consumption or investment in the physical capital stock. The firms hire capital and labor competitively and produce output according to a Cobb-Douglas production technology

$$Y = K^\alpha L^{1-\alpha}$$

where L is aggregate labor input and K is aggregate capital stock. The aggregate resource constraint is

$$C + K' - (1 - \delta)K = Y = K^\alpha L^{1-\alpha} \quad (1)$$

$$\alpha = 1/3 \quad (2)$$

In all the questions below assume that μ (the measure of each cohort) is set such that in a stationary equilibrium $L = 1$

1. Define a stationary equilibrium for this economy
2. Pick h and p to match a cross sectional variance of log earnings of 0.2 and a persistence of individual log earnings of 0.8.
3. Find the values of K and δ such that the stationary equilibrium of this economy displays a return to capital $r = K^{\alpha-1} L^{1-\alpha} - \delta = 1\%$ and a capital output ratio of 3.
4. Set ξ so that the Frisch elasticity of labor supply is $1/2$.
5. Find the value of β for which the economy above is at the stationary equilibrium of point 2 (This point involves actually solving for the stationary equilibrium)
6. Follow a particular cohort in the stationary equilibrium and plot the following as a function of its age: average consumption, dispersion of consumption, wealth and earnings. Briefly explain your findings.