



Economic Growth and Development (8311)

Fall 2006, Mini 2

Problem set 2

Due Tuesday, December 5, in class.

Consider the following Ayiagari type economy. There is continuum of households denoted by wealth a and by individual efficiency units of labor Z who can only trade a non state contingent bond which pays an interest rate r . Preferences are identical for all households and given by

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{c_{t+j}^{1-\gamma}}{1-\gamma}$$
$$\gamma = 2, \beta = 0.99$$

where c is consumption and expectation is taken over possible realizations of future individual efficiency. The budget constraint and borrowing constraint are given by

$$a_t(1+r) + wZ_t \geq c_t + a_{t+1}$$
$$a_{t+1} \geq -\bar{a} \leq 0$$

where \bar{a} is a fixed parameter and w is the wage rate. There is a single good that is being produced in a given period by competitive firms, which can be used for consumption or investment in the physical capital stock K . The representative firms hire capital and labor competitively and produce output according to a Cobb-Douglas production technology

$$Y = AK^\alpha L^{1-\alpha}$$

where $L = \int Z d\phi(Z) = 1$ denotes aggregate labor input which is the sum of individual efficiencies which is assumed to be constant in the aggregate and equal to 1. The aggregate resource constraint is

$$C + K' - (1 - \delta)K = AK^\alpha L^{1-\alpha} \quad (1)$$

$$\alpha = 1/3 \quad (2)$$

Part 1. Calibration of income process

- Assume that you can normalize A such that the equilibrium wage is equal to 1. In this case the process for labor income can be written as $\log Z_t = \rho \log(Z_{t-1}) + \varepsilon_t$. Choose ρ and σ_ε such that you match the following two moments:
 - The mobility matrix in the model's income distribution matches the second largest eigenvalue of the PSID, 1991-1992 income mobility matrix (as constructed in PS 1),
 - The ratio between mean income in the top 10% of the income distribution to mean income in the model matches the same ratio in the 2004 income distribution from SCF (see <http://www.federalreserve.gov/PUBS/oss/oss2/2004/bull0206.pdf>)
- Construct a 5 states Markov Chain that approximates $\log Z_{it}$ using Tauchen method.

Part 2. Decision rules

Assume that the individual's borrowing limit is equal to one year worth of average (cross sectional) income. Guess an interest rate $r = 0.5\%$. Compute decision rules of the individual problem using three methods: policy function iteration, value function derivative iteration, value function derivative iteration with endogenous grid. Report execution times and plot the decision rules for a_{t+1} as a function of a_t for the middle state of your Markov chain.

Part 3. Equilibrium

Define a stationary equilibrium for this economy. Compute the equilibrium interest rate. Calibrate the parameter δ such that the capital income ratio in the economy is equal to 3. Compute the ratio between average wealth in the top 10% of the wealth distribution and average wealth and the ratio between median wealth in the in the top 10% of the wealth distribution and median wealth. Compare these numbers in the model with the equivalent numbers in the SCF. Explore one favourite change of parameters or calibration targets (in the context of the this model) that would bring the model closer to the data.