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Risk Sharing: Private Insurance Markets or Redistributive Taxes?

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ABSTRACT

We explore the welfare consequences of different taxation schemes in an economy where agents are debt-constrained. If agents renege their debts, they are banned from future credit markets, but retain their private (labor) endowments which are subject to income taxation. We impose individual rationality constraints on agents guaranteeing no exclusion in equilibrium and solve for the equilibrium consumption distribution across agents. A change in the tax system changes the severity of punishment from exclusion. We demonstrate that a change to a more redistributive tax system leads to a restriction of the set of contracts that are individually rational and that this restriction leads to a limitation of possible risk sharing via private contracts. The welfare consequences of a change in the tax system depend on the relative magnitudes of increased risk sharing enforced by the new tax system and the reduced risk sharing in private insurance markets. We quantitatively address this issue by calibrating an artificial economy to US income and tax data. We show that for a plausible selection of the structural parameters of our model, the change to a more redistributive tax system leads to *less* risk sharing among individuals and, hence, *lower* ex-ante welfare.

KEYWORDS: Incomplete Markets, Insurance, Limited Enforcement

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1. Introduction

Are redistributive income taxes desirable as a risk sharing device against idiosyncratic income uncertainty? The insights that economic theory provides for this question depend on the assumptions about the structure of private insurance markets. If these markets are complete, in that agents can trade a complete set of perfectly enforceable insurance contracts, then complete risk sharing is achieved via private markets and redistributive income taxes provide no additional insurance. If, on the other hand, private insurance markets are nonexistent or incomplete, redistributive taxes might provide additional insurance. As Mirrlees (1974), Varian (1980), among others, point out, this beneficial effect of redistributive taxes has to be traded off against the adverse effect on incentives to supply labor and to accumulate capital, leading to a nontrivial optimal taxation problem.[2]

Recent empirical studies (see Hayashi et al. (1996), Attanasio and Davis (1996)) have seriously challenged the complete markets assumption, mostly on the ground that risk sharing among individuals is not perfect. As Hayashi et al. conclude: "Our result that there is no full insurance even among related households should serve as a final blow to the complete markets paradigm."

Starting from the empirical observation of incomplete risk sharing, a large body of literature has introduced some form of market incompleteness into the economic environment. Some authors (see Huggett (1993), Aiyagari (1994), among others) assume that some insurance markets are nonexistent for exogenous reasons (usually it is assumed that agents can only self-insure via a single uncontingent bond and face borrowing constraints). The optimal tax analyses by Mirrlees (1974) and Varian (1980) fall into this category.

This modeling strategy does not explicitly capture what we believe is a crucial aspect of redistributive taxation as a risk sharing device: a change in the progressivity of the tax system

[2] A second common justification for redistributive taxation is the social desire to attain a more equal income or wealth distribution. Although we believe that this justification is potentially important we will not address this point in this paper. See Mirrlees (1971) for an analysis of the trade-off between the equity and the labor supply incentive effect of redistributive taxation.

might affect the incentives and opportunities private agents have to engage in private risk-sharing arrangements and thus the form and extent of market incompleteness. To repeat Dixit's (1989) quote of Stiglitz (1981): "Without a clear specification of the information/transactions technology, there is always a danger that any intervention in the economy designed, say, to alleviate problems arising from an absence of risk markets will be either infeasible or so costly to implement that it would not, in fact, constitute a Pareto improvement, for precisely the same reasons that the markets were absent in the first place."

We therefore explore an alternative approach that models the source of incomplete risk sharing *explicitly*.[3] Our approach relies on the assumption of limited enforceability of private contracts.[4] We follow the approach of Kehoe and Levine (1993, 2001) and assume that a complete set of private insurance contracts can be traded. These contracts, however, can not be legally enforced. The only enforcement mechanism for existing contracts is the threat of exclusion from future credit and insurance markets upon default on existing contracts. Tax liabilities, however, are not subject to this enforcement problem as we assume that the penalty for defaulting on tax payments can be made prohibitively large by the government. If agents default on their private debt, they are banned from future credit and insurance markets, but retain their private (labor) endowment which is still subject to income taxation. We impose individual rationality constraints on agents guaranteeing no exclusion in equilibrium. A change in the tax system changes the severity of punishment from default by altering the utility an agent can attain without access to insurance markets. We demonstrate that a change to a more redistributive tax system leads to a restriction of the set of contracts that are individually rational. In an economy that is characterized by uncertainty with respect to individual endowments, this restriction leads to a limitation of possible risk sharing via

[3] Dixit (1989) demonstrates the importance of modeling the source of incomplete risk sharing explicitly for the example of trade policy.

[4] Another fraction of the literature derives market incompleteness from informational frictions underlying the phenomena of adverse selection and moral hazard (see Cole and Kocherlakota (1998) and their review of the literature).

private contracts. The welfare consequences of a change in the tax system then depend on the relative magnitudes of increased risk sharing enforced by the new tax system and the reduced risk sharing in private insurance markets.

We quantitatively address this issue by designing an artificial economy calibrated to US income and tax data. We first measure a stochastic pre-tax income process and a tax system using Consumer Expenditure Survey (CEX) data. We then compare steady state consumption allocations arising under different tax systems.

We find that, for a reasonable selection of the structural parameters of our model, making taxes more progressive leads to *less* risk sharing among individuals, *lower* ex-ante welfare and *higher* consumption inequality. We also show that the opposite happens in an economy in which risk sharing is limited for reasons exogenous to the model. These results, that we view as our main economic contribution, demonstrate that, when analyzing a redistributive tax policy reform, it is crucial to take Stiglitz (1981) seriously and model the underlying source of limited risk sharing explicitly. We want to stress that our result is derived in a model in which the previously mentioned adverse welfare effects of redistributive taxes due to reduced incentives to work and to accumulate capital is completely absent.

On the theoretical side, others have studied economies with debt constraints (see Kocherlakota (1996) and Alvarez and Jermann (2000) among others). These authors, however, consider economies with only two (types of) agents in which household heterogeneity is limited. In a related but independent paper Attanasio and Rios-Rull (2000) use such a model to study the effect of mandatory public insurance programs against aggregate uncertainty on private insurance arrangements against idiosyncratic uncertainty. Although their economy is populated by a large number of (potentially heterogeneous) agents, by assumption agents can only enter pairwise insurance arrangements, not involving any other member of the population. So their underlying insurance problem is equivalent to the ones studied by Kocherlakota and Alvarez and Jermann. Similar to our result they show that the extent to which idiosyncratic shocks can be insured away depends negatively on the public provision of insurance against aggregate uncertainty. Ligon, Thomas and Worrall (2000a, 2000b) set up a model with a finite, but potentially large number of agents that can engage in mutual insurance schemes. Once they solve for constrained-efficient insurance contracts numerically, however, they need to restrict attention to economies with either two agents (as in Ligon, Thomas and Worrall (2000b), which also includes capital accumulation), or they need to assume that agents engage in contracts with the rest of the population, treating the rest of the population as one agent (as in Ligon, Thomas and Worrall (2000a)). This again reduces the problem to a bilateral insurance problem as in the other papers discussed previously.[5]

The main methodological contribution of this work is the analysis of a debt constrained economy with a *continuum* of agents facing idiosyncratic uncertainty: this allows us to analyze insurance mechanisms involving the entire population and not only pairwise relationships. We view this as crucial in our quantitative analysis of risk sharing arrangements such as progressive taxation since gains from insurance are particularly sizable among a large pool of agents with mostly idiosyncratic (i.e. largely uncorrelated) income uncertainty (such as the US labor force). In addition our model, in contrast to the previous literature, endogenously delivers a rich cross-sectional consumption distribution and thus may be of independent interest for the study of other policy reforms where distributional issues are important. But it is also exactly the rich cross-sectional dimension of the model that leads to considerable theoretical and computational complications in solving it. To this end adapt the work of Atkeson and Lucas (1992, 1995) who study efficient allocations in an economy with a continuum of agents and private information. We then show,

[5]The authors have to do so in order to avoid the curse of dimensionality. In their set-up of the problem the cumulative Lagrange multipliers on the enforcement constraints for each agents become (continuous) state variables, in practice ruling out computing allocations for economies with more than two agents.

following Kehoe and Levine (1993), how to decentralize efficient allocations as equilibrium allocations in a standard Arrow Debreu equilibrium with individual rationality constraints.

The paper is organized as follows. In Section 2 we lay out the model environment and define equilibrium. In Section 3 we define and characterize efficient allocations. Section 4 discusses the decentralization. Section 5 presents qualitative features of the equilibrium. Section 6 discusses our policy experiments and Section 7 the parameterization we employ for these experiments. In Section 8 we present our quantitative results, in Section 9 we investigate the sensitivity of our results to parameter changes and in Section 10 we compare our results with those obtained for a standard incomplete markets economy. Section 11 concludes; tables, figures and proofs are contained in the appendix.

2. The Economy

There is a continuum of consumers of measure 1, who have preferences over consumption streams given by

$$U(\{c_t\}_{t=0}^{\infty}) = (1-\beta)E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] \tag{1}$$

The period utility function $u : \Re_+ \rightarrow D \subseteq \Re$ is assumed to be strictly increasing, strictly concave, twice differentiable and satisfies the Inada conditions. Its inverse is denoted by $C : D \rightarrow \Re_+$. Hence $C(u)$ is the amount of the consumption good necessary to yield period utility u. Let $\bar{D} = \sup(D)$; note that we do not assume u to be bounded so that $\bar{D} = \infty$ is possible.

An individual has stochastic endowment process $e \in E$, a finite set with cardinality N, that follows a Markov process with transition probabilities $\pi(e'|e)$. For each consumer the transition probabilities are assumed to be the same. We assume a law of large numbers,[6] so that the fraction of agents facing shock e' tomorrow with shock e today in the population is equal to $\pi(e'|e)$. We assume

[6] Note that we do not require independence of endowment processes across individuals; the assumption of a law of large numbers can then be justified with Feldman and Gilles (1985), proposition 2.

that $\pi(e'|e)$ has unique invariant measure $\Pi(.)$. We denote by e_t the current period endowment and by $e^t = (e_0, .., e_t)$ the history of realizations of endowment shocks; also $\pi(e^t|e_0) = \pi(e_t|e_{t-1}) \cdots \pi(e_1|e_0)$. We use the notation $e^s|e^t$ to mean that e^s is a possible continuation of endowment shock history e^t. We also assume that at date 0 (and hence at every date), the cross-sectional measure over current endowment is given by $\Pi(.)$, so that the aggregate endowment is constant over time. At date 0 agents are distinguished by their initial asset holdings, a_0 (claims to period zero consumption) and by their initial shock e_0. Let Θ_0 be the joint measure of initial assets and shocks.

The government uses taxes to finance a constant amount of public spending g in every period that yields no utility to consumers. The government specifies a tax policy $\tau(e_t)$ that is constant over time. We take government policies $g, \tau(.)$ as exogenously given. For an individual we let $y_t = e_t(1 - \tau(e_t))$ be the after-tax income. Since the function $\tau(.)$ does not depend on time, for a given tax function $\tau(.)$ there is a one-to-one mapping between pre-tax and after-tax endowments. From now on we let $y \in Y \subseteq \Re_{++}$ denote an individual's generic after-tax endowment, following the Markov process π with invariant distribution Π and denote by $y^t = (y_0, \ldots y_t)$ a history of after-tax endowment shocks. We restrict the government policies $g, \tau(.)$ to satisfy

$$g = \int e_t \tau(e_t) d\Pi. \tag{2}$$

With this assumption resource feasibility for this economy states that the sum of all agents' consumption has to be less or equal than the sum over all individuals' after-tax endowment. Therefore, once $g, \tau(.)$ are fixed and hence the after-tax endowment process is specified, we can carry out the subsequent analysis without explicit consideration of the government.

Consumers can trade a full set of state-contingent commodities. A consumption allocation $c = \{c_t(a_0, y^t)\}$ specifies how much an agent of type (a_0, y_0) consumes who experienced a history of endowment shocks y^t. Individuals, at any point in time, have the option to renege on existing contracts. The only punishment for doing so, and hence the only enforcement mechanism for contracts, is that agents that default on their contracts are banned from future insurance markets. They are, however, allowed to self-insure by saving (but not borrowing) at an exogenous constant interest rate r.[7] The expected continuation utility for an agent who defaults after history y^t is given by $U^{Aut}(y_t; r) = U(0, y_t)$, where U is the solution to the functional equation

$$(3) \qquad U(a, y) = \max_{0 \leq a' \leq y+(1+r)a} (1-\beta)u(y + (1+r)a - a') + \beta \sum_{y'} \pi(y'|y) U(a', y')$$

with $a_0 = 0$ given. It is obvious that $U^{Aut}(y_t; r)$ is strictly increasing in y_t, as long as the income shocks are uncorrelated or positively correlated over time.

Individuals have no incentive to default on a consumption allocation c, at any point in time and any contingency, if and only if an allocation satisfies following continuing participation or debt constraints

$$(4) \qquad U_t(a_0, y^t, c) \equiv (1-\beta) \left(u\left(c(a_0, y^t)\right) + \sum_{s>t} \sum_{y^s|y^t} \beta^{s-t} \pi(y^s|y^t) u\left(c(a_0, y^s)\right) \right) \geq U^{Aut}(y_t; r) \qquad \forall y^t$$

i.e. if the continuation utility from c is at least as big as the continuation utility from defaulting on c, for all histories y^t. Since there is no private information and markets are complete, exclusion will not happen in equilibrium as nobody would offer a contract to an individual for a contingency at which this individual would later default with certainty.

Notice that our specification of the debt constraint is more general than the one introduced by Kehoe and Levine (1993) in which agents who default are not allowed to save. If $r = -1$, our model is equivalent to theirs and the right hand side of the debt constraint reduces to

$$(5) \qquad U^{Aut}(y_t; -1) = (1-\beta) \left(u(y_t) + \sum_{s>t} \sum_{y^s|y^t} \beta^{s-t} \pi(y^s|y^t) u(y_s) \right)$$

From now on, whenever there is no danger of ambiguity, we omit the dependence of U_t^{Aut} on r.

[7] This assumption is motivated by current US bankruptcy laws. Agents filing for bankruptcy under Chapter 7 must surrender all their assets above certain exemption levels; the receipts from selling these assets are used to repay the consumer's debt. Remaining debt is discharged. In most cases of Chapter 7 bankruptcy debtors have no non-exempt assets (see White (1998)), so the consequences of filing for bankruptcy only entail restrictions on future credit. Individuals that declared personal bankruptcy are usually denied credit for seven years from major banks and credit card agencies. We view our assumption of being banned forever as a first (and easily tractable) approximation, keeping in mind that it may overstate the punishment from default.

A. Equilibrium

We now define a competitive equilibrium for the economy described above. We will follow the approach of Kehoe and Levine (1993). Consider an agent with period zero endowment of y_0 and initial wealth of a_0. Wealth is measured as entitlement to the period 0 consumption good. Let Θ_0 be the joint distribution over (a_0, y_0) and denote by $p_t(y^t)$ the date zero price[8] of a contract that specifies delivery of one unit of the consumption good at period t to/from a person who has experienced endowment shock history y^t. For each contingency $c_t(a_0, y^t) - y_t$ is the net trade of individual (a_0, y_0) for that contingency. In period 0 there is no uncertainty, so normalize the price of the consumption good at period 0 to 1.

A household of type (a_0, y_0) chooses an allocation $\{c_t(a_0, y^t)\}$ to solve

$$\max U_0(a_0, y_0, c) \tag{6}$$

$$\text{s.t. } c_0(a_0, y_0) + \sum_{t=1}^{\infty} \sum_{y^t|y_0} p_t(y^t) c_t(a_0, y^t) \leq a_0 + y_0 + \sum_{t=1}^{\infty} \sum_{y^t|y_0} p_t(y^t) y_t \tag{7}$$

$$U_t(a_0, y^t, c) \geq U^{Aut}(y_t) \tag{8}$$

Note that, as in Kehoe and Levine (1993), the continuing participation constraints enter the individual consumption sets directly.

Definition 1. *An equilibrium consists of prices* $\{p_t(y^t)\}_{t=0}^{\infty}$ *and allocations* $\{c_t(a_0, y^t)\}_{t=0}^{\infty}$ *such that*

- *given prices, the allocation solves household's problem for almost all* (a_0, y_0)

[8] Note that in standard Arrow Debreu equilibrium theory with finitely many consumers, a complete description of the state of the economy would be *everybody's* endowment shock history, and all prices would be contingent on this complete state. With atomistic individuals, the assumed law of large number and no aggregate uncertainty, attention can be restricted to equilibria in which prices (and quantities) depend only on own personal histories.

- *markets clear, i.e. for all* t,

$$(9) \qquad \int \sum_{y^t} c_t(a_0, y^t)\pi(y^t|y_0)d\Theta_0 = \int \sum_{y^t} y_t \pi(y^t|y_0)d\Theta_0.$$

As is clear from the equilibrium definition our economy does not include physical capital accumulation or government debt, so assets are in zero net supply and the aggregate asset to income ratio is identically equal to zero. While this may seem unrealistic, we deliberately chose to abstract from both types of assets. In a closed economy with incomplete markets and precautionary savings motives an increase in income uncertainty leads to higher precautionary saving, hence higher investment, a higher steady state capital stock and thus higher steady state production (see Aiyagari (1994)). In our economy relaxed borrowing constraints drive the interest rate up and thus, in a version of the model with capital, the aggregate capital stock down. Since in this paper we want to focus on the risk sharing properties of different taxation schemes rather than the effects of taxation and income uncertainty on capital accumulation, we compromise on realism to more clearly isolate the *potential* quantitative importance of the crowding-out mechanism introduced in this paper.

With respect to government debt, the government budget constraint would mandate that, for the same amount of outstanding government debt, the amount of taxes levied to finance the interest payments on the debt would vary across steady states, due to changes in the interest rate. Since the comparison of private households' welfare across economies with different tax burdens seems problematic, we also abstract from government debt in this paper.

3. Efficient Allocations

The next step in our analysis is to characterize and compute equilibrium allocations. Unfortunately this is hard to do by tackling the equilibrium directly. In particular, the presence of the infinite number of dynamic constraints (8) restricting the choice of state contingent claims does not allow to solve the household's problem as a standard dynamic programming problem. Therefore in this section we will follow Atkeson and Lucas (1992, 1995) to first characterize efficient allocations and then argue in the next section that they can be decentralized as competitive equilibrium allocations. As shown by Atkeson and Lucas solving for efficient allocations *does* reduce to solving a standard dynamic programming problem which makes their approach so useful for our problem. As they, however, we also have to restrict our analysis to stationary allocations, i.e. to allocations for which the cross-sectional consumption and wealth distribution is constant over time.

The key insight of Atkeson and Lucas is to analyze the problem of finding efficient allocations in terms of state contingent *utility* promises rather than state contingent consumption. Instead of being indexed by initial assets and endowment shock, now individuals are indexed by initial entitlements to expected discounted utility at period 0, w_0 and initial endowment shocks, y_0. We will discuss the connection between initial asset positions and initial utility promises in Section 4. Let Φ_0 be the period 0 joint measure over (w_0, y_0). An allocation is then a sequence $\{h_t(w_0, y^t)\}_{t=0}^{\infty}$ that maps initial entitlements w_0 and sequences of shocks y^t into levels of current utility in period t. Here $h_t(w_0, y^t)$ is the current period utility that an agent of type (w_0, y_0) receives if she experienced a history of endowment shocks y^t. Note that $c_t(a_0, y^t) = C(h_t(w_0, y^t))$ for an agent whose utility entitlement w_0 corresponds to initial asset holdings a_0, where C is the inverse of the period utility function as defined in Section 2. For any allocation $h = \{h_t(w_0, y^t)\}_{t=0}^{\infty}$ define

$$(10) \quad U_t(w_0, y^t, h) = (1-\beta)\left(h_t(w_0, y^t) + \sum_{s>t}^{\infty} \sum_{y^s|y^t} \beta^{s-t}\pi(y^s|y^t)h_t(w_0, y^s)\right)$$

Equation (10) defines the continuation utility from an allocation h of agent of type (w_0, y_0) from date t and shock history y^t onwards.

Definition 2. *An allocation* $\{h_t(w_0, y^t)\}_{t=0}^{\infty}$ *is constrained feasible with respect to a joint distribution over utility entitlements and initial endowments,* Φ_0*, if for almost all* $(w_0, y_0) \in$*supp*(Φ_0)

$$(11) \quad w_0 = U_0(w_0, y_0, h)$$

$$(12) \qquad U_t(w_0, y^t, h) \geq U^{Aut}(y_t) \qquad \forall y^t$$

$$(13) \qquad \lim_{t\to\infty} \beta^t \sup_{y^t} U_t(w_0, y^t, h) = 0$$

$$(14) \qquad \sum_{y^t} \int \left(C(h_t(w_0, y^t)) - y_t \right) \pi(y^t|y_0) d\Phi_0 \leq 0. \quad \forall t$$

An allocation $\{h_t(w_0, y^t)\}_{t=0}^{\infty}$ *is efficient with respect to* Φ_0 *if it is constrained feasible with respect to* Φ_0 *and there does not exist another allocation* $\{\hat{h}_t(w_0, y^t)\}_{t=0}^{\infty}$ *that is constrained-feasible with respect to* Φ_0 *and such that*

$$(15) \qquad \sum_{y^t} \int C(\hat{h}_t(w_0, y^t)) \pi(y^t|y_0) d\Phi_0 < \sum_{y^t} \int C(h_t(w_0, y^t)) \pi(y^t|y_0) d\Phi_0 \text{ for some } t$$

We call equation (11) the promise keeping constraint: the allocation delivers utility w_0 to agents entitled to w_0. Equations (12) are the continuing participation constraints.[9] Equation (13) is a boundedness condition that assures that continuation utility goes to zero in the time limit. Equation (14) is the resource feasibility condition, requiring aggregate consumption in every period to be less or equal than aggregate endowment in that period. The definition basically says that a utility allocation is efficient if it attains the utility promises made by Φ_0 in an individually rational and resource feasible way and there is no other allocation that does so with less resources. In order to use recursive techniques, however, we have to restrict ourselves to stationary allocations. Define Φ_t to be the joint measure over endowment shocks y_t and continuation utilities $U_t(w_0, y^t, h)$ for a given allocation. An allocation is stationary if $\Phi_t = \Phi_0 = \Phi$. In the next subsections we will show that such an allocation exists, characterize it and demonstrate how to compute it.

A. Recursive Formulation

In order to solve for stationary efficient allocations we consider the problem of a planner that is responsible of allocating resources to a given individual and who can trade resources at a

[9] Note that a Φ_0 that puts positive mass on (w_0, y_0) and satisfies $w_0 < U^{Aut}(y_0)$ does not permit a constraint feasible allocation as promise keeping and debt constraints are mutually exclusive. We restrict attention to Φ_0 with the property that only initial utility entitlements at least as big as the utilty from autarky have positive mass.

fixed intertemporal price $\frac{1}{R}$. In this subsection we discuss such a planners' recursive problem and in the next subsection its solution. We then show that the planners' policy functions induce a Markov process over utility promises and income shocks which has a unique invariant distribution, and finally we demonstrate that there exists an R^* at which the resources needed to deliver utility promises dictated by the stationary distribution equal the aggregate endowment in the economy.

For constant $R \in (1, \frac{1}{\beta}]$, consider the following functional equation (FE) problem. Individual state variables are the promise to expected discounted utility that an agent enters the period with, w, and the current income shock y. The planner chooses how much current period utility to give to the individual, h, and how much to promise her for the future, $g_{y'}$, conditional on her next periods endowment realization y'. We now make the following assumptions on the individual endowment process[10]

Assumption 1: $\pi(y'|y) = \pi(y')$ *for every* $y', y \in Y$

Assumption 2: $\pi(y) > 0$, *for all* $y \in Y$

The operator T_R defining the functional equation of the planner's problem is:

$$
\begin{aligned}
(16)\qquad T_R V(w) &= \min_{h, \{g_{y'}\}_{y' \in Y} \in D} \left\{ \left(1 - \frac{1}{R}\right) C(h) + \frac{1}{R} \sum_{y' \in Y} \pi(y') V(g_{y'}) \right\} \\
(17)\qquad \text{s..t } w &= (1-\beta)h + \beta \sum_{y' \in Y} \pi(y') g_{y'} \\
(18)\qquad g_{y'} &\geq U^{Aut}(y') \qquad \forall y' \in Y
\end{aligned}
$$

where $V(w)$ is the resource cost for the planner to provide an individual with expected utility w when the intertemporal shadow price of resources for the planner is $\frac{1}{R}$. The cost consists of the cost for utility delivered today, $(1 - \frac{1}{R})C(h)$, and expected cost from tomorrow on, $\sum_{y'} \pi(y') V(g_{y'})$, dis-

[10] For the quantitative analysis we will relax these assumptions; however, we could not prove some of our theoretical results without these assumptions.

counted to today. Atkeson and Lucas (1992, 1995) show that a stationary allocation $\{h_t(w_0, y^t)\}_{t=0}^{\infty}$ is efficient if it is induced by an optimal policy from the functional equation above with $R > 1$ and satisfies the resource constraint with equality.[11]

Equation (17) is the promise-keeping constraint: an individual that is entitled to w in fact receives utility w through the allocation rules $\{h(.), g_{y'}(.)\}_{y' \in Y}$. The continuing participation constraints in equation (18) state that the social planner for each state tomorrow has to guarantee individuals an expected utility promise at least as high as obtained with the autarkic allocation. The utility in autarky is given as the solution to the functional equation in (3).

B. Existence and Characterization of Policy Functions for Fixed R

We first prove the existence of optimal allocation rules in the problem with the additional constraints $g_{y'} \leq \bar{w}$ in (18), where $\bar{w}$ is an upper bound on future utility promises. We then characterize the solution of this problem and show that the additional constraints are not binding so that the solution to the problem with additional constraints is also solution to the original problem.[12] The modified Bellman equation is defined on $C(W)$, that is, the space of continuous and bounded functions on W, where $W = \{w \in \Re | \underline{w} \leq w \leq \bar{w}\} \subseteq D$ is a compact subset of $\Re$ and $\underline{w} := \min_y U^{Aut}(y)$. This gives us a standard *bounded* dynamic programming problem. From now on we will denote by T_R the operator defined above, but *including* the additional constraints.

Note that with the additional constraints on future utility promises, (17) and (18) imply that for every w in W possible choices h for current utility satisfy

$$(19) \quad \underline{h}(w) := \frac{w - \beta\bar{w}}{(1-\beta)} \leq h \leq \frac{w - \beta \sum \pi(y') U^{Aut}(y')}{(1-\beta)} =: \bar{h}(w)$$

[11] A policy $(h, \{g_{y'}\})$ induces an allocation, for all (w_0, y_0), in the following way: $h_0(w_0, y_0) = h(w_0, y_0), w_1(w_0, y^1) = g_{y_1}(w_0, y_0)$ and recursively $w_t(w_0, y^t) = g_{y_t}(w_{t-1}(w_0, y^{t-1}), y_t)$ and $h_t(w_0, y^t) = h(w_t(w_0, y^t), y_t)$. Adaptations of their proofs to our environment are contained in Section 1.1 of a separate theoretical appendix, available at http://www.stanford.edu/~dkrueger/theoreticalapp.pdf.

[12] Note that if we had assumed that u and hence C are bounded functions this complication is avoided as the upper bound on u serves as upper bound $\bar{w}$. The results to follow do *not* require boundedness of u.

Accordingly define $\underline{h} := \underline{h}(\underline{w})$ and $\bar{h} := \bar{h}(\bar{w})$. We will show below that we can choose $\bar{w} = \max_y U^{Aut}(y) + \varepsilon$, for $\varepsilon > 0$ arbitrarily small, without the constraints $g_{y'}(w) \leq \bar{w}$ binding at the optimal solution, for all $w \in W$. In order to assure that the constraint set of our dynamic programming problem is compact, for all $w \in W$ we need (since D need not be compact)

Assumption 3: $[\underline{h}, \bar{h}] \subseteq D$.

Assumption 3 is an assumption purely on the fundamentals (u, π, Y, r) of the economy and hence straightforward to check. In particular, for $r = -1$ (the case studied by Kehoe and Levine (1993)) we have $\bar{h}(\bar{w}) = u(y_{\max}) \in D$ and $\underline{h}(\underline{w}) = u(y_{min}) - \beta[u(y_{\max}) - Eu(y)] \in D$ as long as $\frac{y_{\max}}{y_{\min}}$ is sufficiently small and/or β is sufficiently small.[13]

Using standard theory of dynamic programming with bounded returns it is easy to show that the operator T_R has a unique fixed point $V_R \in C(W)$ and that for all $v_0 \in C(W)$, $||T_R^n v_0 - V_R|| \leq \frac{1}{R^n}||v_0 - V_R||$, with the norm being the sup-norm. Also V_R is strictly increasing, strictly convex and continuously differentiable and the optimal policies $h(w), g_{y'}(w)$ are continuous, single-valued functions.[14]

We will now use the first order conditions to characterize optimal policies.

$$
\begin{aligned}
C'(h) &\leq \frac{1-\beta}{\beta(R-1)} V'(g_{y'}) \\
&= \frac{1-\beta}{\beta(R-1)} V'(g_{y'}) \qquad \text{if} \qquad g_{y'} > U^{Aut}(y') \\
w &= (1-\beta)h + \beta \sum_{y' \in Y} \pi(y') g_{y'}
\end{aligned} \tag{20}
$$

The envelope condition is:

$$
V'(w) = \frac{(R-1)}{R(1-\beta)} C'(h) \tag{21}
$$

[13] For CRRA utility with coefficient of relative risk aversion $\sigma \geq 1$ and $r = -1$ assumption 3 is always satisfied.

[14] The proofs of these results are again adaptations of proofs by Atkeson and Lucas (1995). They are contained in Section 1.2. of the separate theoretical appendix at http://www.stanford.edu/~dkrueger/theoreticalapp.pdf.

First we characterize the behavior of h and $g_{y'}$ with respect to w. The planner reacts to a higher utility promise w today by increasing current and expected future utility, i.e. by smoothing the cost over time and across states. The continuing participation constraints, though, prevent complete cost smoothing across different states: some agents have to be promised more than otherwise optimal in certain states to be prevented from defaulting in that state. This is exactly the reason why complete risk sharing may not be constrained feasible.

Lemma 1. Let assumptions 1-3 be satisfied. The optimal policy h, associated with the minimization problem in (16) is strictly increasing in w. The optimal policies $g_{y'}$, are constant in w and equal to $U^{Aut}(y')$ or strictly increasing in w, for all $y' \in Y$. Furthermore

$$
\begin{aligned}
g_{y'}(w) &> U^{Aut}(y') \text{ and } g_{\bar{y}'}(w) > U^{Aut}(\bar{y}') \text{ imply } g_{y'}(w) = g_{\bar{y}'}(w) \\
g_{y'}(w) &> U^{Aut}(y') \text{ and } g_{\bar{y}'}(w) = U^{Aut}(\bar{y}') \text{ imply } g_{y'}(w) \leq g_{\bar{y}'}(w) \text{ and } y' < \bar{y}'
\end{aligned}
$$

Proof. See Appendix

The last part of the lemma states that future promises are equalized across states whenever the continuing participation constraints permit it. Promises are increased in those states in which the constraints bind.

Now we state a result that is central for the existence of an upper bound $\bar{w}$ of utility promises. For promises that are sufficiently high it is optimal to deliver most of it in terms of current period utility, and promise less for the future than the current promises. This puts an upper bound on optimal promises in the long run, the main result in this section, stated in Theorem 1.

Lemma 2. Let assumptions 1-3 be satisfied. For every $(w, y') \in W \times Y$, if $g_{y'}(w) > U^{Aut}(y')$, then $g_{y'}(w) < w$. Furthermore, for each y', there exists a unique $w_{y'}$ such that $g_{y'}(w_{y'}) = w_{y'} = U^{Aut}(y')$.

Proof. See Appendix

THEOREM 1. *Let assumptions 1-3 be satisfied. There exists a* $\bar{w}$ *such that* $g_{y'}(w) < w$ *for every* $w \geq \bar{w}$ *and every* $y' \in Y$.

Proof. See Appendix

Note that the preceding theorem implies that whenever $w \in [\underline{w}, \bar{w}] = W$, then for all $y' \in Y$, the constraint $g_{y'}(w) \leq \bar{w}$ is never binding; since the constraint set in the original dynamic programming problem without the additional constraints is convex, the policy functions characterized in this section are also the optimal policies for the original problem for all $w \in W$. For any $(w_0, y_0) \in W \times Y$ these policies then induce efficient sequential allocations as described in Section A.

The policy functions $g_{y'}$ together with the transition matrix π induce a Markov process on $W \times Y$. In the next subsection we will show that this Markov process has a unique invariant measure, the long-run cross sectional distribution of utility promises (and hence welfare) and income, for any given fixed $R \in (1, \frac{1}{\beta})$.

C. Existence and Uniqueness of an Invariant Probability Measure

Let $\mathcal{B}(W)$ and $\mathcal{P}(Y)$ the set of Borel sets of W and the power set of Y. The function $g_{y'}(w)$, together with the transition function π for the endowment process, defines a Markov transition function on income shock realizations and utility promises $Q : (W \times Y) \times (\mathcal{B}(W) \times \mathcal{P}(Y)) \rightarrow [0, 1]$ as follows:

$$Q(w, y, \mathcal{W}, \mathcal{Y}) = \sum_{y' \in \mathcal{Y}} \begin{cases} \pi(y') & \text{if } g_{y'}(w) \in \mathcal{W} \\ 0 & \text{else} \end{cases} \tag{22}$$

Given this transition function, we define the operator T^* on the space of probability measures $\Lambda((W \times Y), (\mathcal{B}(W) \times \mathcal{P}(Y))$ as

$$(23) \quad (T^*\lambda)(\mathcal{W}, \mathcal{Y}) = \int Q(w, y, \mathcal{W}, \mathcal{Y}) d\lambda = \sum_{y' \in \mathcal{Y}} \pi(y') \int_{\{w \in W | g_{y'}(w) \in \mathcal{W}\}} d\lambda$$

for all $(\mathcal{W}, \mathcal{Y}) \in \mathcal{B}(W) \times \mathcal{P}(Y)$. Note that T^* maps Λ into itself (see Stokey et. al. (1989), Theorem 8.2). An invariant probability measure associated with Q is defined to be a fixed point of T^*. We now show that such a probability measure exists and is unique.

THEOREM 2. *Let assumptions 1-3 be satisfied. Then there exists a unique invariant probability measure Φ associated with the transition function Q defined above. For all $\Phi_0 \in \Lambda((W \times Y), (\mathcal{B}(W) \times \mathcal{P}(Y))$, $(T^*\Phi_0)^n$ converges to Φ in total variation norm.*

Proof. See Appendix

Note that Lemma 2. and Theorem 1. above imply that any ergodic set of the Markov process associated with Q must lie within $[U^{Aut}(y_{\min}), U^{Aut}(y_{\max})] \times Y$ and that the support of the unique invariant probability measure is a subset of this set.

So far we proved that, for a fixed intertemporal price R, policy functions $(h, g_{y'})$, cost functions V and invariant probability measures Φ exist and are unique. From now on we will index $(h, g_{y'}), V$ and Φ by R to make clear that these functions and measures were derived for a fixed R. In the next section we will discuss how to find the intertemporal price R^* associated with an *efficient* stationary allocation. Remember from Subsection A that this requires the allocation to satisfy the aggregate resource constraint with equality, a constraint that we have not yet imposed and will do so in the next subsection in order to solve for R^*.

D. Determination of the "Market Clearing" R

In this section we will analyze how the resource requirements imposed by the cross-sectional distribution of utility promises Φ_R vary with R. We will provide conditions under which an R^* exists for which these resource requirements exactly equal the aggregate endowment in the economy.

In the previous section we showed that for a fixed $R \in (1, \frac{1}{\beta})$ there exists a unique stationary joint distribution Φ_R over (w, y). Define the "excess demand function" $d : \left(1, \frac{1}{\beta}\right) \rightarrow \Re$ as

$$d(R) = \int V_R(w) d\Phi_R - \int y d\Phi_R \tag{24}$$

In this section we discuss the qualitative features of the function $d(.)$. Since by assumption $\bar{y} := \int y d\Phi_R$ does not vary with R, the behavior of d depends on how V_R and Φ_R vary with R. The behavior of Φ_R with respect to R in turn depends on the behavior of $g_{y'}^R$ with respect to R as $g_{y'}^R$ determines the Markov process to which Φ_R is the invariant probability measure. Following Atkeson and Lucas (1995) we can show that $d(R)$ is continuous and increasing on $(1, \frac{1}{\beta})$.[15]

Thus, if one can show that

$$\lim_{R \searrow 1} d(R) < 0 \tag{25}$$

$$\lim_{R \nearrow \frac{1}{\beta}} d(R) > 0 \tag{26}$$

then the existence of a resource-clearing R^* follows.[16]

[15] Again the arguments are adaptations of Atkeson and Lucas' (1995) results and available in Section 1.3 of the separate theoretical appendix at http://www.stanford.edu/~dkrueger/theoreticalapp.pdf. For continuity of $d(R)$ one shows that V_R is uniformly continuous in R and that $g_{y'}^R$ is continuous as a function of R so that Φ_R is continuous in R (in the sense of weak convergence). For monotonicity of $d(R)$ the key results are that $g_{y'}^R$ is increasing in R so that $\Phi_R(., y)$ is increasing in R (in the sense of stochastic dominance), which, together with the fact that V_R is increasing in w proves that $d(R)$ is an increasing function.

[16] Also note that, given our previous theoretical results, it is straightforward to search for R^* numerically: fix an R^0, solve the planners' dynamic programming problem (which we proved to have a unique solution), determine the induced invariant measure over promises (whose existence and uniqueness we proved), and compute $d(R^0)$. If $d(R^0) > 0$, reduce the guess for R, otherwise increase it. We have included details of our computational algorithm in Section 2 of the separate theoretical appendix, available at http://www.stanford.edu/~dkrueger/theoreticalapp.pdf.

The Case $R = \frac{1}{\beta}$

In this subsection we characterize optimal policies of the planner for $R = \frac{1}{\beta}$ and provide a sufficient condition for (26) to hold. Note that for $R = \frac{1}{\beta}$

$$(27) \quad g_{y'}(w) = \begin{cases} w & \text{if } w \geq U^{Aut}(y') \\ U^{Aut}(y') & \text{if } w < U^{Aut}(y') \end{cases}$$

from the first order conditions of the recursive planners' problem (which still has a unique solution as all the results of Section B go through). Now there is a continuum of invariant measures associated with the Markov chain induced by the optimal policies. From (27) it is clear that any such measure $\Phi_{\frac{1}{\beta}}$ satisfies $w \notin \text{supp}\left(\Phi_{\frac{1}{\beta}}\right)$ for all $w < U^{Aut}(y_{\max})$ as the probability of leaving such a w is at least $\pi(y_{\max})$ and the probability of coming back (into a small enough neighborhood) is 0. Therefore all w in the support of any possible invariant measure satisfy $g_{y'}(w) = w$. From the promise-keeping constraint $h(w) = w$ follows, and each individuals' consumption is constant over time: for $R = \frac{1}{\beta}$ there is complete risk sharing.

For complete risk sharing to be efficient it has to satisfy the resource constraint. Since the cost function V_R is strictly increasing in w, the one of the continuum of invariant measures with lowest cost is

$$(28) \quad \Phi_{\frac{1}{\beta}}(w, y) = \begin{cases} \pi(y) & \text{if } w = U^{Aut}(y_{\max}) \\ 0 & \text{if } w \neq U^{Aut}(y_{\max}) \end{cases}$$

All individuals receive utility promises $w = U^{Aut}(y_{\max})$ and hence the same current utility $h(U^{Aut}(y_{\max})) = U^{Aut}(y_{\max})$. This allocation has per-period resource cost $C(U^{Aut}(y_{\max}))$ and is resource feasible if and only if $C(U^{Aut}(y_{\max})) \leq \bar{y}$, or applying the strictly increasing period utility function u to both sides, if and only if $U^{Aut}(y_{\max}) \leq u(\bar{y})$. Let the net resource cost of this allocation be denoted by

$$(29) \quad d\left(\frac{1}{\beta}\right) = C(U^{Aut}(y_{\max})) - \bar{y}$$

We summarize the discussion in the following

LEMMA 3. Let assumptions 1-3 be satisfied. For $R = \frac{1}{\beta}$ any solution to the recursive social planners' problem exhibits complete risk sharing. There exists an efficient stationary allocation with complete risk sharing if and only if $U^{Aut}(y_{\max}) \leq u(\bar{y})$.

Intuitively, the lemma states that it is constrained efficient to share resources equally among the population in this economy if the agents with the highest incentive to renege on this sharing rule, namely the agents with currently high income, find it in their interest to accept constant consumption at $c = \bar{y}$ and lifetime utility $u(\bar{y})$, rather than to leave and obtain lifetime utility $U^{Aut}(y_{\max})$.

Using arguments similar to showing continuity of $d(R)$ on $(1, \frac{1}{\beta})$ one can show that $\lim_{R \nearrow \frac{1}{\beta}} d(R) = d\left(\frac{1}{\beta}\right)$, where $d\left(\frac{1}{\beta}\right)$ is defined as in (29). In order to rule out complete risk sharing[17] we now make

Assumption 4: $U^{Aut}(y_{\max}) > u(\bar{y})$

Note that this assumption is satisfied if the time discount factor β is sufficiently small, agents are not too risk-averse or the largest endowment shock is sufficiently large. We obtain

LEMMA 4. Let assumptions 1-4 be satisfied. Then $\lim_{R \nearrow \frac{1}{\beta}} d(R) > 0$.

Proof. Applying the strictly increasing cost function C to the inequality of assumption 4 gives

$$d\left(\frac{1}{\beta}\right) = C\left(U^{Aut}(y_{\max})\right) - \bar{y} > 0$$

The Case of R Approaching 1

In this subsection we provide necessary and sufficient conditions for autarky (all agents consume their endowment in each period) to be an efficient allocation and characterize policies for R approaching 1 from above.

[17] If there is complete risk sharing under a particular tax system (remember that the tax system maps a given pre-tax income process into a particular after-tax income process), then a small tax reform has no effect on the extent of risk sharing since the resulting allocation is the complete risk sharing allocation: our crowding-out effect is absent.

If agents are very impatient and/or the risk of future low endowments is low, then it is not efficient for the planner to persuade currently rich agents to give up resources today in exchange for insurance tomorrow. For $r = -1$ (no saving after default, as in Kehoe and Levine (1993))this result can be stated and proved formally in the next

LEMMA 5. Let $r = -1$ and let assumptions 1-3 be satisfied. Autarky is efficient if and only if

$$(30) \quad \beta \frac{u'(y_{\min})}{u'(y_{\max})} < 1$$

Proof. For the if-part we note that the autarkic allocation satisfies the first order conditions for some $R > 1$ if (30) holds. Since autarky is constrained feasible, it is efficient.[18] The only-if part is proved in the appendix.

The previous lemma provides a condition under which $d(R) = 0$ as R approaches 1, with autarky as the (efficient) allocation. In order to assure that autarky is not efficient[19] we make

Assumption 5:

$$(31) \quad \beta \frac{u'(y_{\min})}{u'(y_{\max})} \geq 1$$

With assumption 5, as R approaches 1, the resulting allocation features some, but (as long as assumption 4 holds) not complete risk sharing. We state the following conjecture, which we were able to prove for CRRA utility, $r = -1$ and $Y = \{y_l, y_h\}$ but not for the general case considered here.[20]

CONJECTURE 3. *Let assumptions 1-5 be satisfied. Then there exists $R > 1$ such that $d(R) < 0$.*

[18] This is in fact true for arbitrary $r \geq -1$.

[19] If the efficient allocation is autarkic a small change in the tax system changes the allocation on a one to one basis with after tax incomes. No private insurance is crowded out since no private insurance takes place.

[20] Given our other theoretical results, we can check whether $d(R) < 0$ for R sufficiently close to 1 numerically. In all our quantitative experiments this was the case.

We then can conclude our theoretical analysis of stationary efficient allocations with the following theorem, whose proof follows directly from the previous lemmas and conjecture.[21]

Theorem 4. *Let assumptions 1-5 be satisfied. There exists* $R^* \in (1, \frac{1}{\beta})$ *such that* $d(R^*) = 0$. *The allocation induced by* $(h^{R^*}, g_{y'}^{R^*})$ *is efficient and has some, but not complete risk sharing.*

As indicated above, some of our results and proof strategies resemble Atkeson and Lucas (1995). The basic strategy to prove existence of a stationary general equilibrium (as we will show in the next section stationary efficient allocations induce stationary equilibrium allocations) also exhibits similarities to existence proofs for standard incomplete markets models as in Huggett (1993) and Aiyagari (1994).[22] The main difference is that the authors, due to the simple asset structure in their models, can tackle the equilibrium directly. As we do, they first, for a fixed and constant interest rate, solve a simple dynamic programming problem[23] (they for the single household, with assets as state variable, we for the planners, with utility promises as state variables). Then they let the optimal policies induce a Markov process to which a unique invariant distribution is shown to exist.[24] Finally the market clearing interest rate is determined from the goods or asset market clearing condition.[25] These similarities in the theoretical analysis also suggest similar computational algorithms when solving both models numerically.

[21] No claim of uniqueness can be made. In all our numerical exercises $d(R)$ was *strictly* increasing, however, yielding a unique R^* and associated unique stationary efficient allocation.

[22] We will contrast the quantitative findings from our model with the Huggett (1993) version of the standard exogenous incomplete markets model in Section 10.

[23] As in our model, boundedness of the state space for assets from above has to be assured. Huggett assumes that income can only take two values, but doesn't need the stochastic process to be *iid* over time nor any assumption on the period utility function. Aiyagari assumes *iid* income and u to be bounded and to have bounded relative risk aversion -see his working paper. We do not require any boundedness assumption on u, but need the *iid* assumption.

[24] The theorems invoked for the existence of a unique invariant measure are similar in spirit; in particular they all require a "mixing condition" that assures that there is a unique ergodic set. In their setting agents with bad income shocks run down their assets, and good income shocks induce upward jumps in the asset position; in our setting agents with bad shocks move down in the entitlement distribution towards $U^{Aut}(y_{\min})$, with good shocks inducing jumps towards higher w, due to binding participation constraints.

[25] Huggett provides no theoretical properties of the excess asset demand function, in Aiyagari the presence of physical capital, which makes the supply of assets interest-elastic, assures (together with continuity of the asset demand function) the existence of a market-clearing interest rate.

4. Decentralization

In this section we describe how we decentralize a stationary efficient allocation $h = \{h_t(w_0, y^t)\}_{t=0}^{\infty}$ induced by the optimal policies from the recursive planners' problem as a competitive equilibrium as defined in Section 3. Let $\beta^t \pi(y^t|y_0)\mu(a_0, y^t) \geq 0$ be the Lagrange multiplier associated with the continuing participation constraint at history y^t and $P(y^t) = \{y^\tau | \pi(y^t|y^\tau) > 0\}$ be the set of all endowment shock histories that can have y^t as its continuation. Using the first order necessary conditions of the household's maximization problem (6) one obtains

$$\beta \frac{u'\left(c_t(a_0, y^{t+1})\right) \pi(y^{t+1}|y_0)}{u'\left(c_t(a_0, y^t)\right) \pi(y^t|y_0)} = \frac{p_{t+1}(y^{t+1})}{p_t(y^t)} \frac{1 + \sum_{y^\tau \in P(y^t)} \mu(a_0, y^\tau)}{1 + \sum_{y^\tau \in P(y^{t+1})} \mu(a_0, y^\tau)} \tag{32}$$

Obviously, an agent whose participation constraint does not bind at contingency y^{t+1}, following history y^t, faces the standard complete markets Euler equation (as $\mu(a_0, y^{t+1}) = 0$).

Now consider the efficient allocation of utilities $\{h_t(w_0, y^t)\}$ as found in the previous section. Combining the first order condition and the envelope condition from the planners problem we have for an agent that is unconstrained[26] (see (20) and (21)):

$$\frac{1}{R} = \beta \frac{C'(h_t(w_0, y^t))}{C'(h_{t+1}(w_0, y^{t+1}))} \equiv \beta \frac{u'(c_{t+1}(w_0, y^{t+1}))}{u'(c_t(w_0, y^t))} \tag{33}$$

This suggests that the equilibrium prices satisfy (with normalization of $p_0 = 1$)

$$p_t(y^t) = \frac{\pi(y^t|y_0)}{R^t} = p_t \pi(y^t|y_0). \tag{34}$$

with $p_t = R^{-t}$. That is, the price for a commodity delivered contingent on personal histories is composed of two components, an aggregate intertemporal price $p_t = R^{-t}$ and an individual specific, history dependent component, equal to the probability that the personal history occurs.

[26] If no agent is unconstrained we are in autarky and can take $\frac{1}{R} = \beta \frac{u'(y_{\min})}{u'(y_{\max})}$.

Given prices, the initial wealth level that makes the efficient consumption allocation affordable for an agent of type (w_0, y_0) is given by

$$(35) \quad a_0 = c_0(w_0, y_0) - y_0 + \sum_{t=1}^{\infty} \sum_{y^t|y_0} \frac{\pi(y^t|y_0)}{R^t} \left(c_t(w_0, y^t) - y_t \right) = a_0(w_0, y_0) < \infty$$

where the last inequality follows from the fact that the efficient consumption allocation is bounded from above.[27] Finally, the equilibrium consumption allocation corresponding to the efficient allocation is given by[28]

$$(36) \quad c_t(a_0, y^t) = c_t(a_0^{-1}(w_0, y_0), y^t) = C\left(h_t(w_0, y^t) \right).$$

The preceding discussion can be summarized in the following

Theorem 5. *Suppose that $\{h_t(w_0, y^t)\}_{t=0}^{\infty}$ is a stationary efficient allocation (with associated shadow interest rate $R > \frac{1}{\beta}$). Then prices $\{p_t(y^t)\}$ and allocations $\{c_t(a_0, y^t)\}$, as defined in (34) and (36) are an equilibrium for initial wealth distribution Θ_0 derived from Φ_0 and (35).*

Proof. See Appendix

So far we have shown the existence of a stationary equilibrium of our economy and characterized some of its properties. In the next section we illustrate some of its qualitative features.

5. Qualitative Features of the Efficient Allocation

In this section we illustrate some of the qualitative features of the efficient allocation characterized in the section above. To do so we consider a simple numerical example of our economy in which the endowment process can take only two values, $y_l < y_h$ and saving after default is not

[27] Therefore, to decentralize a particular stationary efficient consumption allocation we require a very particular initial distribution over initial assets. In this sense one of the primitives of our model, Θ_0, can't be chosen arbitrarily, which is true in all steady state analyses.

[28] Given that the optimal recursive policy function $h(., y)$ is a strictly increasing function in w, the $h_t(., y^t)$ and hence the $c_t(., y^t)$ are strictly increasing in w_0. Therefore $a_0(., y_0)$ is strictly increasing and thus invertible. We denote its inverse by a_0^{-1}.

permitted ($r = -1$). Figure 1 shows the invariant consumption measure for this example. As shown in the theoretical part the support of the distribution of utility promises w falls into the interval $[U^{Aut}(y_l), U^{Aut}(y_h)]$. With no saving after default it follows that the support of the stationary consumption distribution falls into the interval $[y_l, y_h]$.

We showed that for all $w \in [U^{Aut}(y_l), U^{Aut}(y_h)]$, $g_{y_h}(w) = U^{Aut}(y_h)$, so that all agents with the high income shock arrive tomorrow with the same utility entitlement $w' = U^{Aut}(y_h)$ and thus will consume the same, $C(h(U^{Aut}(y_h)) \leq y_h$. Thus, for agents with high income their history does not matter. As seen from Figure 1, for agents with low income realizations history does matter. With one bad income shock an agents' utility promise falls from $w = U^{Aut}(y_h)$ to $w' = g_{y_l}(w) < w$ and consumption falls to $C(h(w')) < C(h(U^{Aut}(y_h))$. With further low income realizations the agent works herself through the utility entitlement and consumption distribution, until $w = U^{Aut}(y_l)$ and consumption level $C(h(U^{Aut}(y_l))$ is reached. A single good income shock catapults the agent back to consumption level $C(h(U^{Aut}(y_h))$. Note that current income and current consumption of agents are positively correlated, and that there is some, but not perfect risk sharing.

These results are only suggestive for an economy with more than 2 shocks. However, the phenomena of jumping up upon receiving high endowment shocks and stepwise working through the distribution with low endowment shocks is an inherent property of the model. For higher numbers of exogenous states a richer consumption distribution "in the middle" arises. We now parameterize the economy using US data and use it for our quantitative tax exercises.

6. The Policy Experiments

In this section we describe the policy experiments we conduct. It is our goal to investigate how different tax systems affect the steady state distribution of consumption and welfare. In particular we use the model to measure the extent to which public risk sharing mechanisms (i.e. progressive taxes) and private risk sharing mechanisms (i.e. financial markets) help to insulate private consumption from random income fluctuations.

We first use CEX data to obtain a tax system that we identify with the current (progressive) tax system in the US. Keeping the pre-tax income process unchanged we then study how a change in the tax system affects steady state allocations and welfare.

It is important to stress that we have to restrict our analysis to a comparison of steady states. An unexpected change in government policies changes the possible distribution of lifetime expected discounted utilities this economy can attain with given aggregate resources (which remain unchanged). Thus, for a particular agent the promised utility w she entered the period with is not a valid description of her state after the change in fiscal policy anymore. Consequently a method that employs promised expected utility as a state variable cannot be employed to compute transitional dynamics induced by unexpected policy innovations.[29] Steady state welfare comparisons may distort the true welfare effects of policy reforms because it ignores welfare losses (or gains) along the transition path. In economies with capital accumulation higher investment (and thus lower consumption) is required to reach steady states with a higher capital stock and thus higher steady state consumption. In our economy, however, the total amount of resources available for consumption is constant along the transition path and in the new steady state, so that any transition welfare effect due to a change of a physical state variable is absent; we therefore conjecture that the welfare conclusions we derive are robust to an explicit analysis of the transition dynamics induced by a change in fiscal policy.

[29] Technically speaking, after the unexpected change in fiscal policy either some of the promise-keeping constraints are not binding anymore or some of the debt constraints are violated. Any transition analysis in this economy has to tackle the competitive equilibrium directly, where the equilibrium is formulated as a sequential markets equilibrium in the spirit of Alvarez and Jermann (2000).

7. Calibration

First we describe how we set the parameters governing the individual endowment process, together with the government policies (spending and taxes). We then discuss our selection of the preference parameters β, the subjective time discount factor, and σ, the coefficient of relative risk aversion, as we assume that the period utility function is of CRRA-form.

A. Endowment Process

To characterize the Markov chain governing the individual endowment process in the model we need to set the N possible values the endowment e_t can take and estimate the transition matrix $\pi(e_{t+1}|e_t)$. In order to do so we use household level data from the Consumer Expenditure Survey (CEX) for the years 1986-1998. We use CEX income data, whose quality is supposedly inferior to PSID data because the CEX reports also taxes paid by the household members and transfers received, such as welfare and unemployment insurance payments. We try to reduce measurement error by excluding from our sample households classified as incomplete income respondents, as suggested by Nelson (1994). For the same reason we exclude households that report negative total consumption expenditure. Also, since we interpret our pre-tax endowment concept from the model as labor income we exclude households which are solely composed of members that are older than 64 years and families without labor income earners.

The CEX quantity we interpret as e_t, household endowment before taxes, is labor earnings. In the data we measure this entity by the sum of labor earnings, plus a fraction[30] of business and farm income earned by all the members of the household, all divided by the number of adult equivalents[31] in the household.

[30] The fraction of business and farm income we impute to labor income is 0.864 as reported in Diaz Jimenez, Quadrini and Rios Rull (1997).

[31] The number of adult equivalents is defined as in Deaton and Paxson (1994) as the number of households members over age 16 plus .5 times the number of members below age 16.

We pick $N = 5$. The transition matrix $\pi(e_{t+1}|e_t)$ is computed as follows. For any period t in the CEX sample we group households into 5 relative earning classes delimited by 4 uniformly spaced quintiles: the first class is composed by the bottom 20% of the earning distribution in that quarter, the second class by the following 20% and so on. We then search for all households for which we have earning observations in two consecutive periods and compute which relative class they belong to in the second period. Note that the class delimiters are time-dependent in order to take into account of aggregate growth. We repeat this for every period in the sample. Then the probability of transiting from class i to class j is given by the number of households transiting from i to j, divided by the total number of households starting in class i for the entire sample. To set $e_{1,..,}e_N$ we first compute, in each quarter, the median earning within each earnings class. We detrend it by taking its ratio to the overall median earning for that quarter and then set $e_{1,..,}e_N$ equal to the average across quarters of the median (detrended) earnings in each earning class. Tables 1 and 2 show the results of this exercise.

B. Fiscal Policy

To characterize current fiscal policy we need to measure the values of $\tau(e_j), j = 1, \ldots, N$. In the CEX households are asked to report federal, state and local taxes deducted from their last paycheck separately from any additional (not deducted from paycheck) federal, state and local taxes paid. We add to taxes social security contributions and subtract transfers (welfare, unemployment compensation and food stamps). We then set $\tau(e_j)$ equal to the ratio between the total sum of federal, state and local taxes and social security deducted from paycheck, net of transfers, in the j-th earnings class and the total labor income as measured above in the same class. Once the tax policy is set we can compute the implied level of government spending (net of transfers) such that the budget is balanced in every period. The tax policy we will use in our experiments is the average of the tax policies measured in the last four years of the sample (1995-1998) and is reported in table

3. The implied government spending, net of transfers, is equal to $g = 14.9\%$ of total endowment.

C. Preference Parameters

We calibrate the preference parameters (σ, β) so that the solution for the benchmark model delivers an interest rate of 2.9% per year. This was the average real return on high grade municipal bonds[32] over the period 1986-1998. We use this interest since returns on municipal bonds are usually tax-exempt and hence marginal tax rates for interest income need not be specified.

We set σ equal to 2 and then choose β so to match the interest rate. The non-standard part of this exercise is that in a debt constrained economy for a given σ there might be multiple β that deliver the same interest rate. Figure 3 shows the relation between the time discount factor and the interest rate in our calibrated economy: the figure shows that there are three values of β that generate an equilibrium interest rate of 2.9%. To understand the non-monotonic relationship between R and β note that the real interest rate is given by the marginal rate of substitution of an unconstrained agent

$$\frac{1}{R} = \beta \max_{w_0, y^{t+1}} \frac{u'(c_{t+1}(w_0, y^{t+1})}{u'(c_t(w_0, y^t))}$$

For any value of σ if the time discount factor β is sufficiently close to 1 the efficient allocation involves complete risk sharing of idiosyncratic risk, individual consumption is constant, $\frac{u'(c_{t+1}(w_0, y^{t+1})}{u'(c_t(w_0, y^t))}$ is always equal to 1 and the gross interest rate R equals $\frac{1}{\beta}$, hence is decreasing in β.

As β is reduced, complete risk sharing is no longer an equilibrium and there are two effects on the equilibrium interest rate. On one hand there is the standard effect: with a lower β individuals care less about the future, want to save less and an increase in interest rate is needed to restore equilibrium in the credit market. On the other hand a lower β reduces possible risk sharing[33] and

[32] The series for the returns is available on the Economic Report of the President, (2000), Table B-71.

[33] Households with high income realizations discount the future possibility of having low income heavily and thus large transfers from these agents to low-income agents violate the continuing participation constraints.

increases consumption risk faced by agents; this effect induces consumers to save more and thus a reduction in the interest rate is needed to restore equilibrium.

Figure 3 shows that when β is relatively close to 1 the first effect dominates and when β is relatively close to 0 the second effect dominates. Finally if β is sufficiently close to 0 then autarky is the efficient allocation, the interest rate is given by $\frac{1}{R} = \beta \frac{u'(y_{\min})}{u'(y_{\max})}$, and there are no longer changes in the consumption allocation, implying that the interest rate is again decreasing in β.

For a value of $\sigma = 2$ there are three possible values of β ($\beta = 0.02, \beta = 0.024, \beta = 0.967$) consistent with an interest rate of 2.9% . Since the allocation arising under $\beta = 0.02$ is the autarkic allocation[34] we will focus only on the allocation arising under $\beta = 0.024$ and $\beta = 0.967$.

Finally we specify the consequences of default as exclusion from insurance and credit markets. Individuals, however can privately save at interest rate r even after default. We set $r = 2.9\%$, i.e. individuals can save at the equilibrium interest rate after default.

8. Results for Policy Experiments

Before we present our numerical results we define different measures of risk sharing. We define *Total Intermediation* (TI) of risk as one minus the ratio between the standard deviations of log-consumption to log-pre-tax income:

$$TI = 1 - \frac{std(\log(c))}{std(\log(e))}.$$

Note that when $std(\log(c)) = 0$, $TI = 1$; consumption does not vary at all across individuals and the economy exhibits complete risk sharing. If $std(\log(c)) = std(\log(e))$, $TI = 0$ and consumption varies one to one with pre-tax endowments For $0 < TI < 1$ there is some, but not complete risk sharing, with higher TI indicating higher risk sharing.

[34] The quantitative feature of the autarkic allocation are very similar to those of the allocation arising with a discount factor of $\beta = 0.024$

We can decompose TI into two components reflecting risk intermediation enforced by the government (GI) via the tax system and risk intermediation achieved in addition by private insurance contracts, (PI). We define

$$GI = 1 - \frac{std(\log(y))}{std(\log(e))} \qquad PI = 1 - \frac{std(\log(c))}{std(\log(y))}$$

When $std(\log(y)) = std(\log(e))$, $GI = 0$. After-tax income is as variable as pre-tax income, which is the case if the tax system is proportional. If $std(\log(y)) < std(\log(e))$, $GI > 0$ and the tax system is progressive, with the extreme of complete redistribution via the tax system, $std(\log(y)) = 0$ and $GI = 1$. On the other hand, if $std(\log(y)) > std(\log(e))$, $GI < 0$ and the tax system is regressive. The interpretation of PI is similar: if $std(\log(c)) = 0$, $PI = 0$ and there is complete risk sharing achieved through private markets. If, on the other hand $std(\log(c)) = std(\log(y))$, $PI = 0$ and private markets do not achieve any risk sharing over and above that achieved by the tax system. Note that $TI = GI + (1 - GI) * PI$. Hence total intermediation of risk equals government intermediation of risk plus private intermediation of that part of risk that is not already removed by the tax system. In particular, under a proportional tax system $GI = 0$ and $TI = PI$. We will report the measures TI, GI and PI for all our policy simulations.

As described above we consider the following policy experiment: change the tax system from the progressive system found in the calibration section to a proportional system (constant average tax rates), keeping the level of government spending constant. We compare interest rates, measures of intermediation of risk, welfare (in income units) [35] and consumption inequality (as measured by the Gini index) in the steady states arising under the two tax systems. The results of our policy experiments are summarized in Table 4. For the high value of $\beta = 0.967$ a switch from a progressive

[35] To measure welfare we first determine which tax regime yields higher ex-ante welfare, where ex-ante welfare is measured as $\int h(w, y) d\Phi = \int w d\Phi$. We then increase the after-tax endowment of every agent by $x\%$ in the tax regime with lower welfare and report that x for which ex-ante welfare in the dominating tax regime coincides with that in the dominated tax regime in which $x\%$ of resources have been added.

to a proportional tax system leads to a reduction of publicly enforced risk sharing from 19.7% to 0. But risk sharing achieved via private arrangements increases with the proportional system since the value of autarky falls and hence the debt constraints are relaxed. This effect is sufficiently strong to raise total risk sharing with proportional taxes over the level achieved with progressive taxes. Therefore ex-ante welfare increases by 0.1% with such a tax reform. It is worth noting that consumption inequality is reduced by this change, showing that rich households are not the only beneficiaries from the increase in welfare. In particular households with low income now face less binding borrowing constraints and can increase their consumption borrowing more. In summary, the government, in trying to help households to share risk by making the tax system more progressive, achieves exactly the opposite -*lower* risk sharing and welfare. Again, this result is obtained in a model in which progressive taxes do not distort the labor-leisure decision.

Also note that the interest rate rises with the switch to the proportional system. This is because proportional taxes lower the value of default, hence relaxing the debt constraints. Households can now borrow more and the interest rate has to rise to bring credit markets back into equilibrium.

The results from our policy experiment are drastically different for the case of low time discount factors. If agents discount the future very heavily, the extent of risk sharing achievable with self-enforceable private contracts is very limited, as the threat of future exclusion from credit markets is not severe for very impatient agents. We see from the table that the low risk sharing allocation is significantly different from the high risk sharing allocation. The effect of the tax reform on government intermediated risk sharing is similar to the case with high discount factor (by construction). Now, however, private markets are almost completely ineffective in providing risk sharing for both tax systems. Hence total intermediation of risk consists (almost) exclusively of GI. This explains the large welfare losses going from a progressive to a proportional system. In this case consumption inequality rises as a consequence of the change in the tax system.

Again the interest rate rises (this time very sharply) with the proportional system. Comparing the high risk sharing with the low risk sharing case we conclude that when private markets are very effective in providing risk sharing contracts, then the attempt of the government to do even more may be counterproductive. If, on the other hand, private markets do not work because it is difficult to enforce private contracts then public risk sharing provided by redistributive taxation leads to potentially large welfare gains.

9. Sensitivity Analysis

In this section we explore the sensitivity of our quantitative findings to changes in the parameter values. In particular we consider changes in the risk aversion coefficient, in the persistence of the income process and alternative changes in the tax system. Throughout this section we focus on the high risk sharing case, corresponding to a $\beta = 0.967$.

A. Risk Aversion and Persistence

We first compute the impact on risk sharing (measured as total intermediation) of the same change in the taxation schemes analyzed in the section above. The first two rows of table 5 we consider economies with a risk aversion parameter of 2.5 (High Risk Aversion) and 1.5 (Low Risk Aversion). In the third and fourth rows we analyze economies with different persistence of the income process. In particular we consider a modified transition matrix for the endowment process $\tilde{\Pi} = (1-\alpha)\Pi + \alpha I$ where I is the identity matrix[36]. We consider two cases: $\alpha = 0.1$ (High persistence) and $\alpha = -0.1$ (Low Persistence).

The results in the table show how the higher the risk aversion and the lower the persistence of the income process the higher the level of risk sharing, since the value of autarky is reduced. Note, however, that in all cases reported a change from proportional to progressive increases total

[36] We adopt this way of modifying the persistence of the income process from Alvarez and Jermann (2000).

risk sharing of the economy, meaning that the increase in private intermediation more than offsets the reduction in public risk sharing. The results also suggests that if risk aversion is sufficiently high, exclusion from credit markets is a harsh punishment and complete risk sharing through private financial markets is achieved, independently of the tax system. In this case, obviously, a change in the tax system would have no effect on consumption allocations.

B. Tax Policies

Instead of focusing on a discrete change in the tax system we now analyze how total risk sharing evolves with a marginal change in progressivity. We restrict our discussion to a tax system of the form

$$\tau(e_j) = a - \frac{b}{e_j} \qquad j = 1, \ldots 5$$

i.e. to a system with constant marginal tax rate a and a fixed deduction b. We estimate a and b by running a regression on our five data points for taxes derived from CEX data. We find estimates $\hat{a} = 25\%$ and $\hat{b} = 9.6\%$. The R^2 of this regression equals a very high 0.995, so that the progressive tax system used in the last section is almost perfectly approximated by a tax system with a constant marginal tax rate of 25% and a fixed deduction of 9.6% of mean per adult earnings (see Figure 4). The implied level of government spending equals 15.5% of average pre-tax earnings, which is similar to the level we used in the previous section. We now vary the degree of progressivity marginally by marginal changes in the deduction b, with varying a correspondingly to assure that all tax systems generate the same revenue[37]. Note that for $b = 0$ we are back in the proportional tax system from the last section. In Figure 5 we plot total risk sharing against the deduction b, where total risk sharing is measured as TI. Complete risk sharing in this economy can be achieved with two rather extreme policies. One policy that obviously achieves the first best allocation is to tax all income

[37] Normalizing the endowments so that $\sum_{j=1}^{5} \Pi(e_j)e_j = 1$ one obtains that to keep the tax revenues constant requires to set $a = g + b$.

differences away, i.e. making the system extremely progressive and equalize after-tax income of all agents. This is achieved by taxing income at a 100% rate ($a = 1$) and by setting the deduction equal to the average income ($b = (1 - g) = \bar{y}$). One the other hand, making the tax system sufficiently regressive makes the punishment from default sufficiently harsh and allows private financial markets to achieve perfect risk sharing. This occurs for $b < 0.1$, i.e. for a poll tax of at least 10% of mean pre-tax earnings. In between these extreme cases the effect on total risk sharing (and hence ex-ante welfare) of a marginal increase in tax progressivity depends on the relative magnitudes of the two effects at work: the direct effect of reducing the variability of after-tax income and the indirect crowding-out effect.

For a fixed deduction of $b < 0.39$ the crowding-out effect dominates; more progressivity reduces total risk sharing and, therefore, ex-ante welfare. Notice that this is a large range of tax policies; furthermore the experiment considered in the previous section falls into this class. For $b > 0.39$ the direct effect dominates and more progressive taxes lead to more total risk sharing among individuals and higher welfare. Overall the welfare differences between the worst tax regime ($b = 0.39$) and first best amounts to about 1% of average income.

10. Incomplete Markets

In this section we contrast our findings with the welfare effects of the same reform in a standard incomplete markets model. In this economy agents are only allowed to trade a single uncontingent bond and they face an exogenously specified constant borrowing limit. There are no enforcement problems in this economy. The economy we consider is similar to the one studied by Huggett (1993). The household problem in recursive formulation reads as

$$v(a, y) = \max_{-\underline{b} \leq a' \leq y + Ra} (1 - \beta)u(y + Ra - a') + \beta \sum_{y'} v(a', y')\pi(y'|y)$$

where a are holdings of the one-period bond and $R-1$ is the interest rate on these bonds. Again we compare stationary equilibria under different tax systems.

To enable comparison with the debt constrained economy we calibrate this economy to the same observations. In particular, the endowment and tax processes are kept the same. The coefficient of relative risk aversion is again chosen to equal $\sigma = 2$. We then identify pairs of borrowing limits $\underline{b}$ and time discount factors β such that the equilibrium interest rate equals 2.9%. Table 6 presents the results for our policy experiments for a selection[38] of $(\underline{b}, \beta)$-pairs

Note that a switch from a progressive to a proportional tax system induces large welfare losses and increases in consumption inequality. Redistributive taxes act as a partial substitute for private insurance markets that are exogenously assumed to be missing from this economy. Removing this substitute for private markets has negative welfare consequences. This is true regardless of how tight the borrowing limit is specified. In contrast to the debt-constrained economy a tax reform (by assumption) does not change the assets that can be traded nor the extent to which they can be traded. Therefore the crowding-out effect that was crucial in the previous sections cannot occur in this economy.

Finally note that while in the debt-constrained economy a shift from progressive to proportional taxes causes an increase in the interest rate, in this economy the interest rate falls. Remember that the tax change increases income risk. In this economy agents thus increase precautionary saving and this excess saving reduces the interest rate.

[38] There is a limit as to how negative one can chose $\underline{b}$. This limit is given by the constraint that agents with $y = y_{\min}$ and $a = -\underline{b}$ can attain nonnegative consumption by setting $a' = -\underline{b}$ at the equilibrium interest rate (for both tax systems). This limit on $\underline{b}$ turns out be approximately equal to 7. Since the average before tax income is normalized to 1 $\underline{b}$ can be interpreted as the ratio between the maximum borrowing limit and the average annual pre-tax earning.

11. Conclusion

In this paper we present a model that highlights a new channel through which income taxation affects private financial markets. Although in our model taxes do not directly affect labor-leisure and wealth accumulation decisions they affect the functioning of private financial markets by changing the incentives to default on private contracts. We show that when private insurance markets are active, public risk sharing provided via taxes crowds out private risk sharing. In order to gain some insights into the magnitude of this effect we calibrate our very simple model to US income and tax data in order to quantify the effects of changes in the progressivity of taxation. We find that the magnitude of the crowding-out effect depends crucially on the level of risk sharing that is achieved via private contracts. When risk sharing provided by private markets is low, crowding out is small and redistribution through taxes is welfare improving, while in high risk sharing regimes redistribution through taxes crowds out private financial markets more than one to one, and therefore is welfare reducing, even if the tax reform does not have adverse incentive effects on the labor supply decision.

In contrast, if private insurance markets are assumed to be missing for model-exogenous reasons, a tax reform that reduces the variance of after-tax income serves as a partial substitute for private insurance markets and leads to unambiguous welfare gains, absent labor supply incentive distortions. This indicates that the assumption about the structure of private capital markets is crucial when analyzing redistributive government policies.

In order to isolate the effect of the tax system on private insurance markets as clearly as possible we abstracted from several features of actual economies that are potentially important. A comprehensive analysis of redistributive taxation should include the effects we highlight in this work, together with the effects of a redistributive tax reform on incentives to work and on physical capital accumulation. We view this as interesting issue for future research.

References

[1] Aiyagari, R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659-684.

[2] Alvarez, F. and U. Jermann (1998): "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, 68, 775-798.

[3] Atkeson, A. and R. Lucas (1992): "On Efficient Distribution with Private Information," *Review of Economic Studies*, 59, 427-453

[4] Atkeson, A. and R. Lucas (1995): "Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance," *Journal of Economic Theory*, 66, 64-88.

[5] Attanasio, O. and S. Davis (1996): "Relative Wage Movements and the Distribution of Consumption," *Journal of Political Economy*, 104, 1227-1262.

[6] Attanasio, O. and V. Rios-Rull (2000): "Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare?," *European Economic Review,* 44, 1225-1258.

[7] Cole, H. and N. Kocherlakota (1998): "Efficient Allocations With Hidden Income and Hidden Storage," *Federal Reserve Bank of Minneapolis, Staff Report* 238.

[8] Deaton, A. and C. Paxson (1994): "Intertemporal Choice and Inequality," *Journal of Political Economy*, 102, 437-467.

[9] Díaz-Giménez, J., V.Quadrini and J.V. Ríos-Rull (1997): "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth", *Federal Reserve Bank of Minneapolis Quarterly Review*, 21, 3-21.

[10] Dixit, A. (1989): "Trade and Insurance with Adverse Selection," *Review of Economic Studies,* 56, 235-248.

[11] Feldman, M. and C. Gilles (1985): "An Expository Note on Individual Risk without Aggregate Uncertainty," *Journal of Economic Theory,* 35, 26-32.

[12] Hayashi, F. , J. Altonji, and L. Kotlikoff (1996): "Risk Sharing Between and Within Families," *Econometrica*, 64, 261-294.

[13] Huggett, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of economic Dynamics and Control*, 17, 953-969.

[14] Kehoe, T. and D. Levine (1993): "Debt Constrained Asset Markets," *Review of Economic Studies*, 60, 865-888.

[15] Kehoe, T. and D. Levine (2001): "Incomplete Markets versus Debt Constrained Markets," forthcoming in *Econometrica.*

[16] Kocherlakota, N. (1996): "Implications of Efficient Risk Sharing without Commitment," *Review of Economic Studies*, 63, 595-609.

[17] Ligon, E., J. Thomas and T. Worrall (2000a), "Mutual Insurance and Limited Commitment: Theory and Evidence in Village Economies," *mimeo*, University of California at Berkeley.

[18] Ligon, E., J. Thomas and T. Worrall (2000b), "Mutual Insurance, Individual Savings, and Limited Commitment," *Review of Economic Dynamics*, 3, 1-47.

[19] Mirrless, J. (1971): "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175-208.

[20] Mirrlees, J. (1974): "Notes on Welfare Economics, Information and Uncertainty, in *Essays on Economic Behavior under Uncertainty,* edited by M. Balch, D. McFadden and S. Wu, Amsterdam: North Holland.

[21] Nelson, J. (1994): "On Testing for Full Insurance Using Consumer Expenditure Survey Data," *Journal of Political Economy*, 102, 384-394.

[22] Stiglitz, J. (1981): "Pareto Optimality and Competition," *Journal of Finance,* 36, 235-251.

[23] Stokey, N. and R. Lucas , with E. Prescott (1989): "Recursive Methods in Economic Dynamics," Harvard University Press, Cambridge.

[24] Varian, H. (1980): "Redistributive Taxation as Social Insurance," *Journal of Public Economics*, 14, 49-68.

[25] White, M. (1998): "Why it Pays to File for Bankruptcy: A Critical Look at Incentives under U.S. Bankruptcy Laws and A Proposal for Change," *mimeo.*

Appendix

Proof of Lemma 1.:

We want to show that for all $\underline{w} \leq w < \hat{w} \leq \bar{w}$, $h(w) < h(\hat{w})$. Suppose not. Then from (20) $V_R'(g_{y'}(w)) \geq V_R'(g_{y'}(\hat{w}))$ for all y' such that $g_{y'}(\hat{w}) > U^{Aut}(y')$, and hence $U^{Aut}(y') < g_{y'}(\hat{w}) \leq g_{y'}(w)$ for all those y' by strict convexity of V_R. From promise keeping there must exist $\bar{y}'$ such that $g_{\bar{y}'}(w) < g_{\bar{y}'}(\hat{w}) = U^{Aut}(\bar{y}')$, a violation of the debt constraint.

Now, since h is strictly increasing in w, $C'(h(w)) < C'(h(\hat{w}))$. Suppose that $g_{y'}(w) > U^{Aut}(y')$. Then from (20) we have $V_R'(g_{y'}(w)) < V_R'(g_{y'}(\hat{w}))$ and from the strict convexity of V_R it follows that $g_{y'}(\hat{w}) > g_{y'}(w)$. Obviously, if $g_{y'}(w) = U^{Aut}(y')$ then $g_{y'}(\hat{w}) \geq g_{y'}(w)$.

Thus we conclude that either $g_{y'}(\hat{w}) > g_{y'}(w)$ or $g_{y'}(w) = g_{y'}(\hat{w}) = U^{Aut}(y')$ ■

Proof of Lemma 2.:

V_R is strictly convex and differentiable. By assumption $g_{y'}(w) > U^{Aut}(y')$. Combining (20) and (21) we obtain $\beta R V_R'(w) = V_R'(g_{y'}(w))$. Since $R < \frac{1}{\beta}$ we have $V_R'(w) > V_R'(g_{y'}(w))$. By strict convexity of V_R the first result follows. Hence $g_{y'}(.)$ are always strictly below the 45^0 line in its strictly increasing part. But $g_{y'}(w) \geq U^{Aut}(y')$ for all w. Hence for $w < U^{Aut}(y')$ it follows that $g_{y'}(w) = U^{Aut}(y') > w$. By continuity of $g_{y'}(.)$ we obtain that $g_{y'}(U^{Aut}(y')) = U^{Aut}(y')$, and from the first result it follows that $g_{y'}(w) < w$ for all $w > U^{Aut}(y')$ ■

Proof of Theorem 1.:

Take $\bar{w} = \max_y U^{Aut}(y) + \varepsilon$, for $\varepsilon > 0$. If $g_{y'}(w) > U^{Aut}(y')$, then the previous Lemma yields the result. If $g_{y'}(w) = U^{Aut}(y')$, then $g_{y'}(w) = U^{Aut}(y') \leq \max_y U^{Aut}(y) < \bar{w}$

Proof of Theorem 2.:

We first prove that there exists $w^* \in W$ such that $w^* > U^{Aut}(y_{\max})$ and $g_{y_{\max}}(w^*) = U^{Aut}(y_{\max})$, from which it follows that $g_{y_{\max}}(w^*) = U^{Aut}(y)$ for all $w \leq w^*$.

Suppose, to obtain a contradiction, that $g_{y_{\max}}(w) > U^{Aut}(y_{\max})$ for all $w \in W, w > U^{Aut}(y_{\max})$.

Then by Lemma 1. we have $g_{y'}(w) = g_{y_{\max}}(w)$, for all $y' \in Y$ and all $w > U^{Aut}(y_{\max})$. By continuity of $g_{y'}$ and Lemma 2. we conclude that $g_{y'}(U^{Aut}(y_{\max})) = U^{Aut}(y_{\max})$, for all $y' \in Y$. But since $U^{Aut}(y_{\max}) > U^{Aut}(y')$ for all $y' \neq y_{\max}$, by Lemma 2. it follows that $g_{y'}(U^{Aut}(y_{\max})) < U^{Aut}(y_{\max})$ for all $y' \neq y_{\max}$, a contradiction.

We now can apply Stokey et al., Theorem 11.12. For this it is sufficient to prove there exists an $\varepsilon > 0$ and an $N \geq 1$ such that for all $(w, y) \in (W, Y)$ we have $Q^N((w, y, U^{Aut}(y_{\max}), y_{\max}) \geq \varepsilon$.

If $w^* \geq \bar{w}$ this is immediate, as then for all $(w, y) \in (W, Y)$, $Q((w, y, U^{Aut}(y_{\max}), y_{\max}) \geq \pi(y_{\max})$, since $g_{y_{\max}}(w) = U^{Aut}(y_{\max})$ for all $w \in W$. So suppose $w^* < \bar{w}$. Define

(A1) $$d = \min_{w \in [w^*, \bar{w}]} \{w - g_{y_{\max}}(w)\}$$

Note that d is well-defined as $g_{y_{\max}}$ is a continuous function and that $d > 0$ from Lemma 2. Define

(A2) $$N = \min\{n \in N | \bar{w} - nd \leq w^*\}$$

and $\varepsilon = \pi(y_{\max})^N$. Suppose an individual receives $y_{\max}$ for N times in a row, an event that occurs with probability ε. For (w, y) such that $w \leq w^*$ the result is immediate as for those $w, g_{y_{\max}}(w) = U^{Aut}(y_{\max})$ and $g_{y_{\max}}(U^{Aut}(y_{\max})) = U^{Aut}(y_{\max})$. For any $w \in (w^*, \bar{w}]$ we have $g_{y_{\max}}(w) \leq w - d$, $g_{y_{\max}}(g_{y_{\max}}(w)) \leq w - 2d$, etc. The result then follows by construction of (N, ε) ■

Proof of Lemma 5.: We first show that there is an allocation attaining a distribution of utility that stochastically dominates the utility distribution in autarky and requires no more resources. It is then immediate that autarky is not efficient. In autarky the measure over utility entitlements and endowment shocks is given by

(A3) $$\Phi^{Aut}(\{U^{Aut}(y), y\}) = \pi(y)$$

We show that there exist allocations that attain the joint measure $\hat{\Phi}$ defined as

$$\hat{\Phi}(\{U^{Aut}(y), y\}) = \pi(y) \qquad \text{all } y \neq y_{\min}$$

$$\hat{\Phi}(\{U^{Aut}(y_{\min}), y_{\min}\}) = \pi(y_{\min})(1-\pi(y_{\max}))$$

(A4) $$\hat{\Phi}(\{\tilde{w}, y_{\min}\}) = \pi(y_{\min})\pi(y_{\max})$$

where $\tilde{w} = U^{Aut}(y_{\min}) + \varepsilon$ for small $\varepsilon > 0$. Define $\delta_{\max}$ and $\delta_{\min}$ implicitly by

$$\tilde{w} = (1-\beta)(u(y_{\min}) + \delta_{\min}) + \beta \sum_{y} \pi(y) U^{Aut}(y)$$

$$U^{Aut}(y_{\max}) = (1-\beta)(u(y_{\max}) - \delta_{\max}) + \beta \sum_{y \neq y_{\min}} \pi(y) U^{Aut}(y) + \beta\pi(y_{\min})\tilde{w}$$

(A5)

Since $\tilde{w} = U^{Aut}(y_{\min}) + \varepsilon$, we have

$$\delta_{\max} = \frac{\beta\pi(y_{\min})}{(1-\beta)}\varepsilon$$

(A6) $$\delta_{\min} = \frac{\varepsilon}{(1-\beta)}$$

The autarkic allocation exhausts all resources. The new allocation reduces consumption for the $\pi(y_{\max})$ agents with $y_{\max}$ by $\delta_{\max}$ and increases consumption for $\pi(y_{\max})\pi(y_{\min})$ agents by $\delta_{\min}$. Hence, compared to the autarkic allocation the change in resource requirements is given by

$$\Delta = -\pi(y_{\max})C'(u(y_{\max}))\delta_{\max} + \pi(y_{\max})\pi(y_{\min})C'(u(y_{\min}))\delta_{\min}$$

(A7) $$= \frac{\pi(y_{\min})\pi(y_{\min})\varepsilon}{(1-\beta)} \left(\frac{-\beta}{u'(y_{\max})} + \frac{1}{u'(y_{\min})} \right)$$

Therefore $\Delta \leq 0$ if and only if

(A8) $$\beta \frac{u'(y_{\min})}{u'(y_{\max})} \geq 1$$

Under this condition the new allocation is resource feasible, incentive feasible and attains $\hat{\Phi}$, a distribution that dominates Φ^{Aut}. It is straightforward to construct the sequential allocation h induced by the recursive policies supporting $\hat{\Phi}$. By reducing $h_0(U^{Aut}(y_{\min}), y_{\min})$ so that the agents receiving discounted utility $\hat{w}$ under $\hat{\Phi}$ receive $U^{Aut}(y_{\min})$ the new allocation attains Φ^{Aut} but with less resources, a contradiction to the assumption that autarky is constrained efficient. ■

Proof of Theorem 5.

The allocation satisfies the resource constraint (9) since the efficient allocation does and Θ_0 is derived from Φ_0. Also the allocation satisfies the continuing participation constraints, and, by construction of $a_0(w_0, y_0)$, the budget constraint. It remains to be shown that $\{c_t(a_0, y^t)\}$ is utility maximizing among the allocations satisfying the budget and the continuing participation constraints. The first order conditions

$$\text{(A9)} \quad (1-\beta)\beta^t \pi(y^t|y_0) u'\left(c_t(a_0, y^t)\right)\left(1 + \sum_{y^\tau \in P(y^t)} \mu(a_0, y^\tau)\right) = \lambda(a_0, y_0) p(y^t)$$

are sufficient for consumer optimality.[39] Define Lagrange multipliers $\mu(a_0, y_0) = 0$, $\lambda(a_0, y_0) = (1-\beta)u'(c_0(a_0, y_0))$ and recursively

$$\text{(A10)} \quad 1 + \sum_{y^\tau | y^t} \mu(a_0, y^\tau) = \frac{u'(c_0(a_0, y_0))}{(\beta R)^t u'(c_t(a_0, y^t))}$$

Note that the allocation by construction (see 33) satisfies $\frac{u'(c_t(a_0,y_0))}{\beta R u'(c_{t+1}(a_0,y^{t+1}))} \geq 1$, with equality if the limited enforcement constraint is not binding. Hence $\mu(a_0, y^{t+1}) \geq 0$ and $\mu(a_0, y^{t+1}) = 0$ if the constraint is not binding. By construction the allocation and multipliers satisfy the first order conditions. ■

[39] The consumer maximization problem is a strictly convex programming problem (the constraint set with the debt constraints remains convex). Note that since the efficient consumption allocation is bounded from above, the expected continuation utility from any history y^T onward, discounted at market prices R^{-T} goes to zero as $T \to \infty$ (i.e. the relevant transversality condition is satisfied). For details see Section 1.4 of the separate theoretical appendix.at http://www.stanford.edu/~dkrueger/theoreticalapp.pdf.

Table 1. Endowment values

Ratio to the median (1986-1998, 89504 obs.)

e_1	e_2	e_3	e_4	e_5
0.18	0.60	1.00	1.51	2.55

Table 2. Transition matrix $\pi(e_{t+1}|e_t)$

(1986-1998, 27633 obs.)

	e_{t+1}				
	0.67	0.18	0.05	0.02	0.02
	0.22	0.57	0.18	0.05	0.02
e_t	0.06	0.19	0.56	0.18	0.03
	0.03	0.05	0.18	0.60	0.15
	0.02	0.02	0.03	0.15	0.78

Table 3. Tax rates

(1995-1998, 26333 obs.)

$\tau(e_1)$	$\tau(e_2)$	$\tau(e_3)$	$\tau(e_4)$	$\tau(e_5)$
-40%	+8.5%	+14%	+16.5%	+19%

Table 4. Results for benchmark parameters

	High Risk Sharing (β=0.967)		Low Risk Sharing (β=0.024)	
	Progressive	Proportional	Progressive	Proportional
Real Rate (R-1)	2.9%	3.1%	2.9%	117%
Intermediation				
Government GI	19.7%	0.%	19.7%	0%
Private PI	92.3%	94.5%	0.2%	0.6%
Total TI	93.8%	94.5%	19.9%	0.6%
Consumption Gini	0.028	0.025	0.349	0.383
Ex Ante Welfare		+0.1%	+26%	

Table 5. Total intermediation in various economies

	Tax System	
	Progressive	Proportional
High Risk Aversion	97.4%	100%
Low Risk Aversion	89.7%	90.5%
High Persistence	92.4%	93.4%
Low Persistence	94.6%	95%
Benchmark	93.8%	94.5%

Table 6. Results for bond economies

	$\underline{b}$=1, β=0.85		$\underline{b}$=4, β=0.93		$\underline{b}$=7, β=0.95	
	Prog.	Prop.	Progr.	Prop.	Prog.	Prop.
Real Rate (R-1)	2.9%	-1%	2.9%	1.8%	2.9%	2.0%
Intermediation						
Government GI	19.7%	0%	19.7%	0%	19.7%	0%
Private PI	15.5%	24.8%	42.5%	38.4%	44.6%	42.2%
Total TI	32.1%	24.8%	42.5%	38.4%	44.6%	42.2%
Consumption Gini	0.22	0.25	0.15	0.16	0.13	0.14
Ex Ante Welfare	+8.9%		+4.6%		+3.5%	

Figure 1. Consumption Distribution

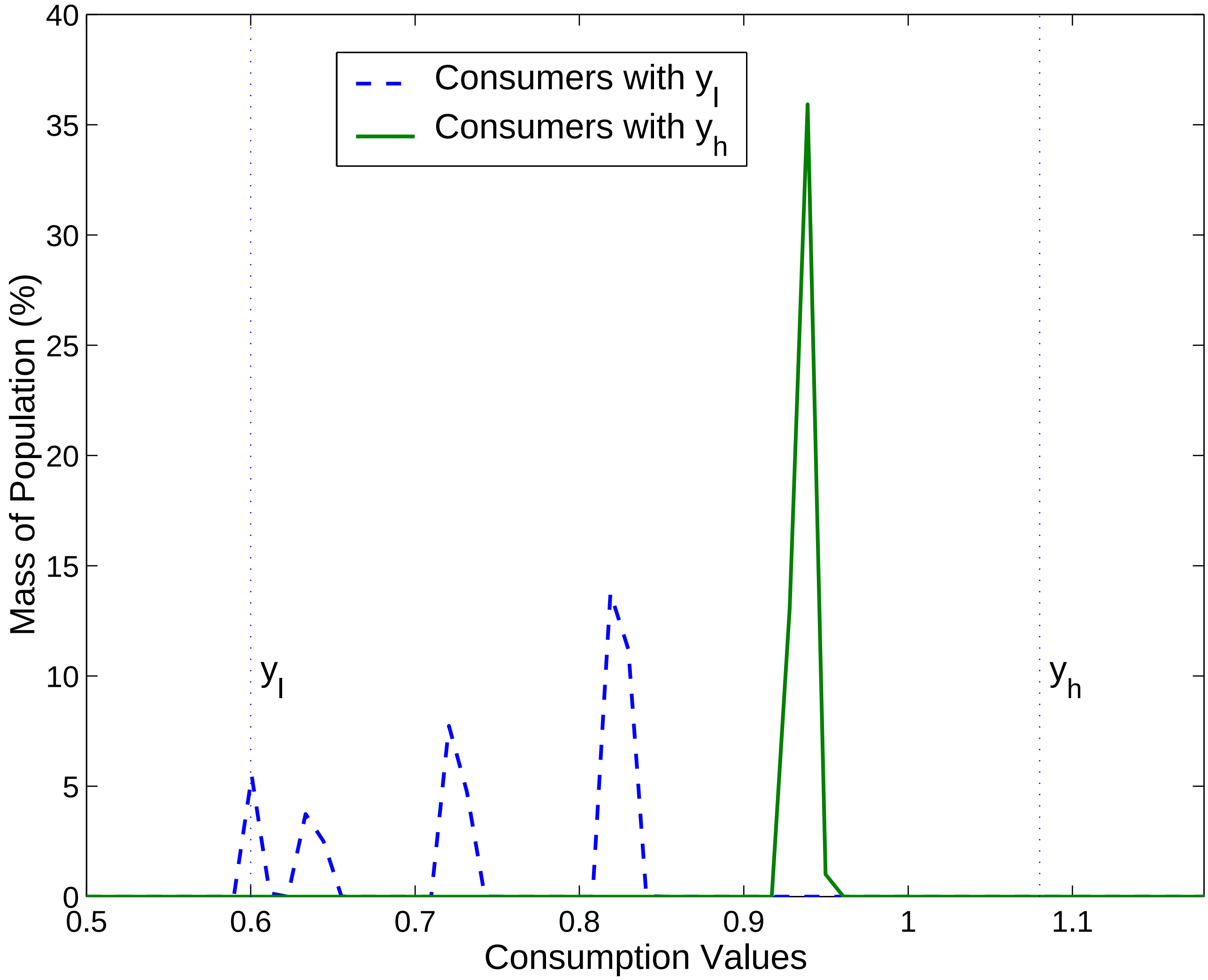

Figure 2. Relation between β and Real Interest Rate

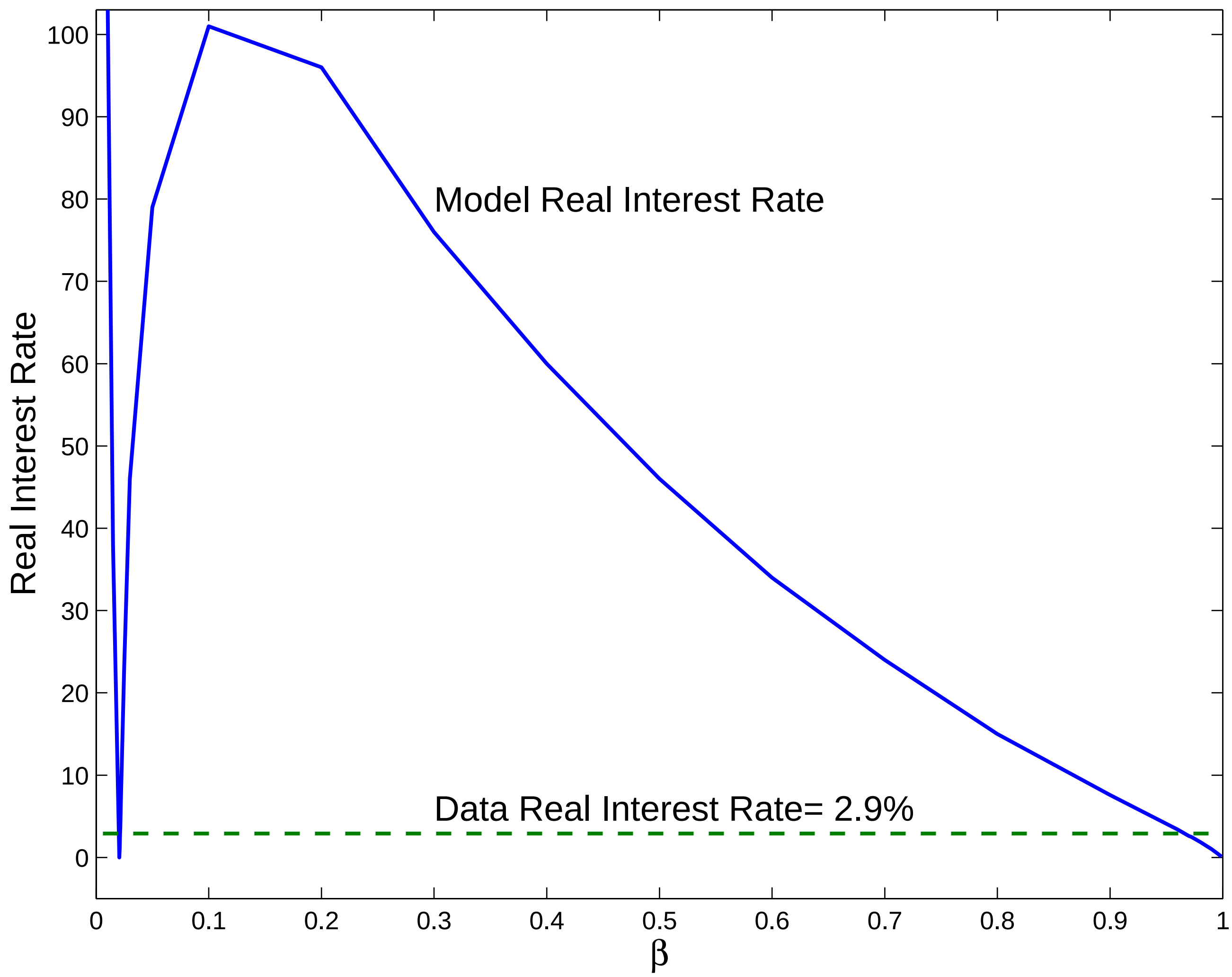

Figure 3. Average 1995−98 Tax Rates (Computed from CEX data)

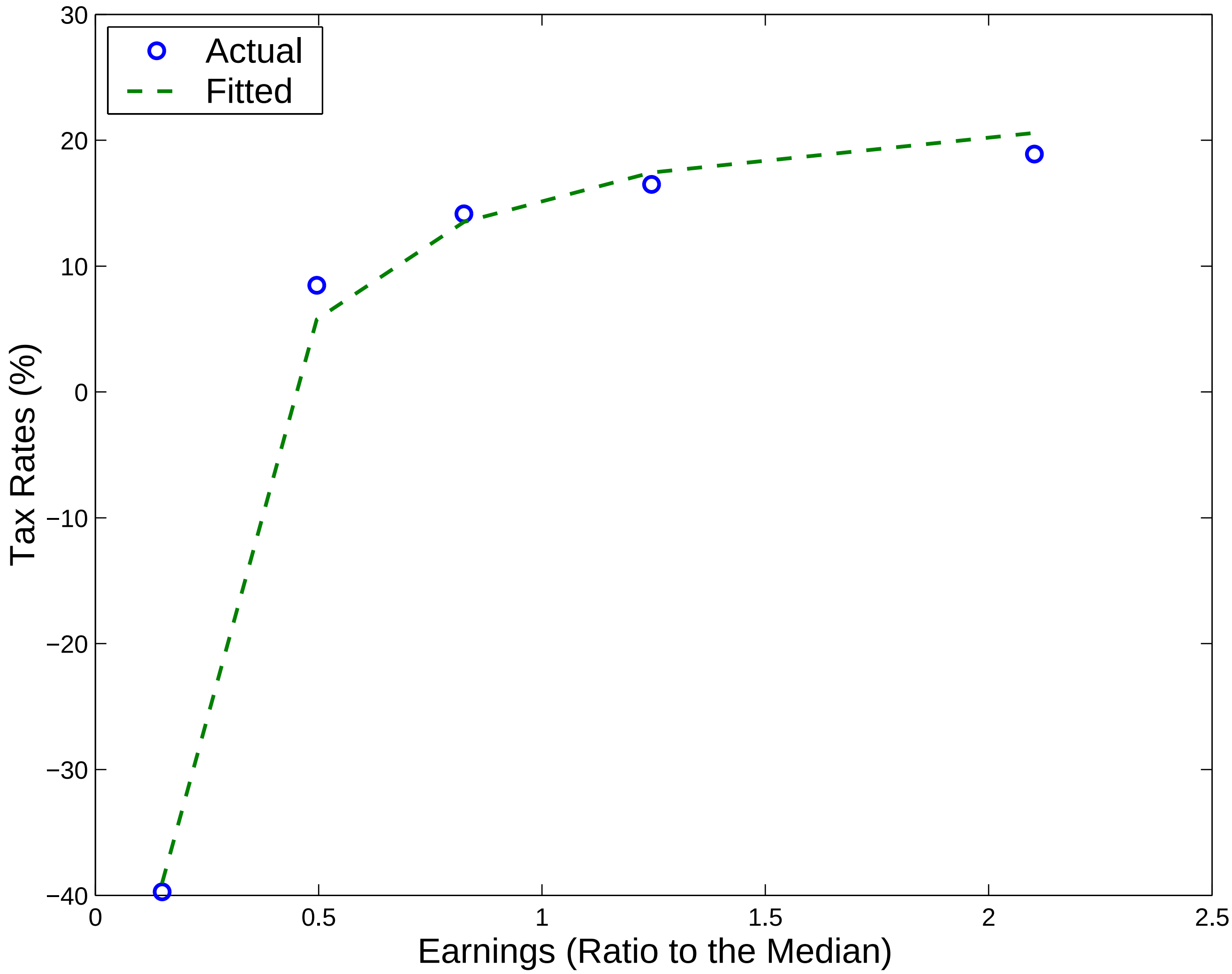

Figure 4. Risk Sharing as a Function of Progressivity of the Tax System