Implications of Asset Market Data for Equilibrium Models of Exchange Rates

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Outline

- summary and intuition
- an alternative resolution
- current events

Goal and result

- What can we learn about financial markets from exchange rates? (kind of the reverse of the title ☺)

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- 2 countries, 2 currencies
- complete markets, 2 bonds traded in both countries
- Exchange rates comovement with SDF suggest wedges in bond trading (preference for home bonds)

Primitives

$$M(z_t, z_{t+1}), M^*(z_t, z_{t+1}), \frac{S_{t+1}(z_{t+1})}{S_t}$$

- $M(z_t, z_{t+1})$,$M^*(z_t, z_{t+1})$ are SDFs, tell how much a home consumer values a dollar in state z_{t+1} relative to state z_t, and how much a foreign consumer values a Euro in state z_{t+1} rel. to z_t.
- An example is

$$M(z_t, z_{t+1}) = \beta \frac{u'(c_{t+1}(z_{t+1}))}{u'(c_t(z_t))}$$

- $S(z_{t+1})$ spot exchange rate in $t+1$ (Euros per USD, so when $S(z_{t+1}) \uparrow$ dollar appreciates)
- All log normally distributed

Complete markets

- Agents in both countries can trade full set of Arrow securities in home and foreign currency
- Let $P(z_t, z_{t+1})$ be the state z_t \$ price of an Arrow security that delivers 1 \$ in state z_{t+1} (and $P^*(z_t, z_{t+1})$ be the Euro price of a Euro in state z_{t+1})
- Euler equations (for each z_{t+1})

$$\begin{aligned} P(z_t, z_{t+1}) &= M(z_t, z_{t+1}) \\ P^*(z_t, z_{t+1}) &= M^*(z_t, z_{t+1}) \end{aligned}$$

- No arbitrage for each z_{t+1} (two ways of delivering a Euro in z_{t+1})

$$\frac{P(z_t, z_{t+1})}{S(z_{t+1})} = \frac{P^*(z_t, z_{t+1})}{S_t}$$

- substituting away the prices of the Arrow securities, and taking logs

$$\Delta s(z_{t+1}) = m(z_t, z_{t+1}) - m^*(z_t, z_{t+1})$$

Are exchange rates consistent with complete markets?

- $$\Delta s(z_{t+1}) = m(z_t, z_{t+1}) - m^*(z_t, z_{t+1})$$
- In states where home consumers value dollar a lot, dollar appreciates! Is this consistent with data?
- If $m(z_t, z_{t+1}) = \log(\beta) - \sigma \Delta c(z_{t+1})$
- $$\Delta s(z_{t+1}) = -\sigma(\Delta c(z_{t+1}) - \Delta c^*(z_{t+1}))$$
- Home currency should appreciate when home has weaker consumption growth than foreign.
- Million papers show that this is not the case (starting with Backus-Smith, 1993)

Risk sharing interpretation

- Under complete markets MRT between Dollars and Euros is equated with MRS state by state
- $\Delta s(z_{t+1})$ change in MRT,
- $-\sigma(\Delta c(z_{t+1}) - \Delta c^*(z_{t+1}))$ change in MRS,
- If valuation of dollars at home raises relative to the valuation of Euros abroad (increase in MRS), it must be that dollars are getting expensive (increase in MRT)
- Conclusion: exchange rate comovement with consumption growth strike against complete markets and perfect risk sharing

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- Conclusion: exchange rate comovement with consumption growth strike against complete markets and perfect risk sharing
- Note that this restriction implies that foreign bonds are bad hedge for domestic agents. If home currency appreciates in bad domestic states, foreign bonds have low payoffs in bad states

2 bonds economy

- Each consumer trade a dollar and a Euro risk free bond, with rates r_t r_t^*
- Interest rate differentials (consistent with Euler equations)

$$r_t^* - r_t = \underbrace{E_t \Delta s(z_{t+1})}_{\text{Compensation for Euro Depreciation}} - \underbrace{\frac{1}{2} Var_t(\Delta s(z_{t+1}))}_{\text{Discount for extra return due to risk (Lognorm)}} - \underbrace{cov_t(m(z_t, z_{t+1}), -\Delta s(z_{t+1}))}_{\text{Hedging Discount}}$$

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- Because both bonds are traded in both countries the first three terms of the eqs above sum to 0,

$$\begin{aligned} Var_t(\Delta s(z_{t+1})) &= cov_t(m(z_t, z_{t+1}), \Delta s(z_{t+1})) - cov_t(m^*(z_t, z_{t+1}), \Delta s(z_{t+1})) \\ &= cov_t(m(z_t, z_{t+1}) - m^*(z_t, z_{t+1}), \Delta s(z_{t+1})) > 0 \end{aligned}$$

- exchange rate is (conditionally) countercyclical!

Some intuition

- Euler equations imply that foreign v/s home interest rate differential compensate expected benefits of going short home v/s long foreign
- No arbitrage implies that the sum of interest rate differentials is 0
- Hence sum of expected benefits is 0
- Lognormality implies that both agent have a positive expected benefit of investing in the foreign risk asset, hence the foreign currency must have an expected cost that offsets this benefit, which is the bad hedging properties, i.e. positive conditional covariance $cov_t(m(z_t, z_{t+1}) - m^*(z_t, z_{t+1}), \Delta s(z_{t+1})) > 0$
- Note that this is similar to the CM restriction
- At a broad level reminiscent of several results that few bonds can dynamically almost complete markets

Final findings

- Paper shows that positive conditional covariance, given other feature of exchange rates, also implies positive unconditional covariance
- Hence a 2 bond economy not consistent with exchange rates
- What market structure can be consistent with observed exchange rates?
- Bond trading with wedges!
- In particular wedges (not necessarily time varying) that increase benefits of holding domestic bonds
- These wedges offset extra benefit of holding foreign bonds due to log-normal risk, poor hedging properties are not needed, hence conditional, and unconditional, covariance can be positive
- $$Var_t(\Delta s(z_{t+1})) = \phi_t + cov_t(m(z_t, z_{t+1}) - m^*(z_t, z_{t+1}), \Delta s(z_{t+1})) > 0$$

An alternative lesson from exchange rates

- The positive covariance between exchange rate changes and SDF is inconsistent with data under the assumption that SDFs are only driven by consumption growth
- Consider the case of complete markets and taste shocks

$$m(z_t, z_{t+1}) = \log(\beta) - \sigma \Delta c(z_{t+1}) + \xi_{t+1}$$

- In this case:

$$\Delta s(z_{t+1}) = -\sigma(\Delta c(z_{t+1}) - \Delta c^*(z_{t+1})) + \xi_{t+1} - \xi^*_{t+1}$$

- correlation between $\Delta s(z_{t+1})$ and $\Delta m(z_{t+1})$ is still 1, however the correlation between $\Delta s(z_{t+1})$ and $\Delta c(z_{t+1}) - \Delta c^*(z_{t+1})$ can become negative, solving the Backus-Smith puzzle
- So wedges in IM or taste shocks IN CM?
- Full answer possibly requires a more structural model of the SDF

Concluding thoughts

- Really enjoyed reading this paper, new and cool results about the connection between exchange rates and bond markets, which I think is a super relevant question
- Focus of the paper is on understanding unconditional moments exchange rates and SDF
- Next important step (not just for this paper) is to understand implications of particular exchange rate development for SDF and financial markets!