International Trade
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International Trade (8402)

Spring 2012, Mini 1

Problem set 2

Due on Friday March 23. Please submit an individual pdf file with output via-email to me. Output should not exceed 7 pages.

Question 1. Consider the standard symmetric 1 good international real business cycle model where households in both countries can trade in stocks.

Domestic households objective

$$\max E \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} c_t^{1-\sigma} - \phi \frac{l_t^v}{v} \right)$$

Foreign households are exactly symmetric. The domestic firms objective is:

$$\begin{aligned} & \max E Q_t d_t \\ d_t &= A_t k_{t-1}^{\alpha} l_t^{1-\alpha} - w_t l_t - x_t \\ k_t &= (1-\delta) k_{t-1} + x_t - \eta k_{t-1} \left(\frac{x_t}{k_{t-1}} - \delta \right)^2 \end{aligned}$$

where Q_t is price uses by firms to value dividends in different states/dates. Assume that $Q_t = U_{c,t}$ i.e. that domestic firms care only about domestic households, regardless of the equilibrium stock holdings. Foreign firms are exactly symmetric. Budget constraints are

$$\begin{aligned} c_t + \lambda_t^D p_t + \lambda_t^F p_t^* &= w_t l_t + \lambda_{t-1}^D (p_t + d_t) + \lambda_{t-1}^F (p_t^* + d_t^*) \\ c_t^* + \lambda_t^{*F} p_t^* + \lambda_t^{*D} p_t &= w_t^* l_t^* + \lambda_{t-1}^{*D} (p_t + d_t) + \lambda_{t-1}^{*F} (p_t^* + d_t^*) \end{aligned}$$

where

λ_t^D Share of domestic stock held by domestic households
λ_t^F Share of foreign stock held by domestic households
λ_t^{*D} Share of domestic stock held by foreign households
λ_t^{*F} Share of foreign stock held by foreign households

Assume that A_t and A_t^* follow independent AR(1) process with persistence parameter $\rho = 0.95$ and standard deviation of innovations equal to 0.01. Assume that $\sigma = 2, \beta = 0.99, \alpha = 0.3, \delta = 0.025, \nu = 2$.

1. Calibrate ϕ and η so that in the complete market version of the model the average labor is 0.3 and investment is 3 times as volatile as GDP.

2. Compute the average equilibrium portfolio holdings of foreign assets. In particular do the following. Guess a steady state portfolio (for example $\lambda_t^D = \lambda_t^{*F} = 1, \lambda_t^F = \lambda_t^{*D} = 0$). Around this guess solve for decision rules approximated up to the second order. Note that in order to do so you will have to first impose a small cost of holding portfolio different from the steady state (i.e. impose a resource cost $\xi(\lambda_t - \lambda_0)^2$, where ξ is a small number and λ_0 is your initial guess) and then use one of the packages available. One package which works well is the one developed by Stephanie Schmitt-Grohe & Martin Uribe, in the paper "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function" (MATLAB programs are available on their web-page but you'll need the symbolic toolbox), alternatively you can use DYNARE. Simulate your model for a number of periods and then check whether the average stock holding in the simulation are similar to the your initial guess of the steady state. If they are not update your initial guess until convergence.

3. Check that the portfolio solution does not change as you locally change ξ

4. Compare your numerical solution with the solution method you would get applying the Devereux and Sutherland method to this model (Country Portfolios in Open Economy Models, Journal of the European Economic Association, April 2011)

5. Assess how average portfolio changes as you change the correlation of the innovation of the shocks. In particular graph the average share of foreign assets held in equilibrium as a function of the correlation of the shocks with the correlation going from 0 to 0.5

Question 2. Consider a 1 period model in which an agent allocates a fixed amount of real wealth W between two assets paying stochastic real returns R_1 and R_2, and has real labor income Y. With CRRA utility the problem can be written as

$$\max_{\alpha} \frac{1}{1-\gamma} E[C^{1-\gamma}]$$
$$C = (R_1\alpha + R_2(1-\alpha))\, W + Y$$

where R_1,R_2 and Y are stochastic variables.Assume that R_1 and R_2 have the same distribution with mean $\bar{R}$ and variance σ and that Y has mean $\bar{Y}$ and variance σ_Y.Assume all stochastic variables are log normally distributed and let lower case letters denote lod deviations of variables from their mean.

1. Log linearize the budget constraint
2. Solve analytically for α as a function of the the ratio $f = \frac{W}{W/+\bar{Y}}$, of γ and of $var(r_1 - r_2)$ and of the covariance of $r_1 - r_2$ with r_2 and y

Question 3. Consider a small open economy as in Arellano (2007). Preferences of the representative agent living there are given by the standard

$$E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

Income can take only two states $y = \{y_l, y_h\}$.The transition probability is given by

$$\Pi = \begin{bmatrix} \pi_l & 1-\pi_l \\ 1-\pi_h & \pi_h \end{bmatrix}$$

The agent starts with 0 wealth, can trade a non contingent bond and can default on it. If the agent defaults she's excluded from credit markets forever. The bond is also traded by risk neutral international investors who face a risk free rate of r. Write down the agent's problem and define equilibrium. Solve numerically for the equilibrium interest rate on bonds and for the solution of the individual's problem. Find a parameters configuration under which in the computed equilibrium the country defaults with positive probability in finite time. Find a path of shocks in which default happens. Along that path plot allocations (c and b) and interest rates and compare them with those that would emerge if the country was not allowed to default.

Question 4. Consider now a two agent economy in which preferences are the same and the income is given by the same process above (each income is an independent process). Agents trade with each other a full set of contingent assets. Agents can default on those assets but if they do so they are in autarky forever. Find a parameter configuration under which the Arrow-Debreu complete markets allocation is not enforceable. For those parameters compute the constrained efficient allocation. Simulate the model and compare the correlation between consumption and income in each country under compete markets and in the constrained efficient allocation.