Macroeconomic Theory
Fabrizio Perri

Macroeconomic Theory (8107)

Spring 2012, Mini 1

Problem set 3

Due Thursday, March 8, in class

Question 1. Income fluctuation problem with exponential utility

Consider a consumer with the following utility function

$$E\sum_{t=0}^{\infty}\beta^t u(c_t)$$
$$u(c) = -\frac{1}{\gamma}e^{-\gamma c}$$

which faces the following income process

$$\begin{aligned} y_t &= \rho_0 + \rho_1 y_{t-1} + \varepsilon_t \\ \varepsilon_t &\rightarrow N(0,\sigma) \\ \rho_0 &> 0, 0 < \rho_1 < 1 \end{aligned}$$

The budget constraint can be written as

$$a_{t+1} = (1+r)a_t + y_t - c_t$$

where $r > 0$ is the interest rate. Assume that in period 0 the consumer starts with assets $a_0 > 0$ and given income $y_0 > 0$.

1. Define the value of the income stream of the consumer (in terms of today's wealth a_t)
$$P_t = \frac{1}{1+r}E_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j y_{t+j}$$
solve for P_t as a function of y_t,ρ_0, ρ_1 and r (show your calculations)

2. Let

$$\begin{aligned} V(a,y) &= \max_c u(c) + \beta E V(a',y') \\ & \quad s.t. \\ a' &= (1+r)a + y - c \end{aligned}$$

be the value function of the consumer. Show that if the value function has the following form

$$\begin{aligned} & \frac{1}{\gamma r} e^{-\gamma r(a+By+D(r))} \\ B &= \frac{1}{1+r-\rho_1}, D(r) \text{ a fixed function} \end{aligned}$$

then the consumption function has the following form

$$c = r(a + By + D(r) + \frac{1}{\gamma r}\log(1+r))$$

3. Show that $D(r)$ has the following form

$$\frac{\rho_0 B}{r} - \frac{1}{\gamma r^2}\Big(\log(\beta(1+r) + \log E(e^{-\gamma r B\varepsilon})) - r\log(1+r)\Big)$$

4. Now consider the special case in which $\rho_1 = 0$ (i.i.d. shocks).Show that $\Delta c_t = c_{t+1} - c_t$ is equal to

$$\Delta c_t = \frac{r}{1+r}(y_{t+1} - \rho_0) + \frac{1}{\gamma}\Big(\log(\beta(1+r) + \log E(e^{-\gamma r B\varepsilon})\Big)$$

5. Show that if $\beta(1+r) = 1$ then individual consumption is a random walk with positive drift (i.e. $c_t = c_{t-1} + \Delta + \Gamma\varepsilon_t$) and solve for Δ.

Question 2. Income and consumption in the PSID

In 2005 the Panel Study of Income Dynamics (PSID) has started collecting comprehensive consumption expenditure data for a panel of US families. Using data from the PSID (http://psidonline.isr.umich.edu/) do the following.

1. Collect data for family labor income (i.e the sum of labor income of the head, the spouse and of others) and family consumption expenditures (in this variable you should include food, household furnishings and equipment, clothing and apparel, trips and vacations, recreation, health care, transportation, education and child care) for 2007 and 2009.

2. Deflate the three measures using the CPI and divide each family measure by the number of members in the family so to obtain per-capita real measures. Compute changes of real per capita income and consumption for each family.

3. Select only families which report positive measures of both quantities in both years and rank families by per-capita real labor income changes. Divide the ranked families in 10 quantiles (i.e. starting from the ones with lowest growth to the one with the highest growth) and produce the following plot: on the x axis report the average change of labor income in each decile and on the y axis report the average (in the group) change in consumption expenditure.

4. Interpret your results using the PIH model studied in class. In particular what does the consumption response tell you about the persistence of the labor income shocks?

Question 3. Consider a closed economy inhabited by a continuum of infinitely lived agents (each agent is indexed by i) with preferences

$$E \sum_{t=0}^{\infty} \beta^t \left(\log(c_{it}) - \frac{l_{it}^{\phi}}{\phi} \right)$$
$$1 > \beta > 0, \phi > 1$$

where c_{it} is consumption (of a single good) and l_{it} is labor. Agents have no initial wealth, and in each period they are endowed with labor productivity e_{it} given by

$$e_{it} = \exp(\varepsilon_{it} + A_i)$$

where

$$A_i \rightarrow N\left(\frac{-\sigma_A^2}{2}, \sigma_A^2\right) \textit{ i.i.d.} \text{ across agents, constant through time}$$
$$\varepsilon_{it} \rightarrow N\left(\frac{-\sigma_\varepsilon^2}{2}, \sigma_\varepsilon^2\right) \textit{ i.i.d.} \text{ across agents and time}$$

Agents can trade a full set of Arrow securities contingent on the realization of their ε and can sell their effective labor $e_{it}l_{it}$ to competitive firms who use a constant returns to scale technology that transforms one unit of effective labor into one unit of consumption good. Let w be the price of one unit of effective labor in terms of the consumption good. There is no capital nor aggregate uncertainty in the economy. Assume throughout that appropriate law of large numbers apply, so that for each period and for each possible value of the permanent shock A, the measure of agents experiencing temporary shocks in a given set is constant.

1. Denote with $p_{\varepsilon'}(A, \varepsilon)$ and $b_{\varepsilon'}(A, \varepsilon)$ the price and the quantity purchased (by an agent experiencing shocks A, ε) of an Arrow security paying off 1 unit of consumption tomorrow if the agent tomorrow experiences shock ε', and with $c(A, \varepsilon), l(A, \varepsilon)$ consumption and labor supply of the same agent. Define a competitive equilibrium for this economy

2. Show that in equilibrium
$$p_{\varepsilon'}(A, \varepsilon) = \beta\pi(\varepsilon')$$
where $\pi(\varepsilon')$ is the pdf of ε',
$$c(A, \varepsilon) = c(A)$$
i.e. consumption is independent from the temporary shock ε, and
$$b_{\varepsilon'}(A, \varepsilon) = c(A) - wl(A, \varepsilon') \exp(\varepsilon' + A)$$

3. Solve for the risk free rate and for the wage per unit of effective labor in this economy

4. Solve for $c(A)$ and for $l(A, \varepsilon)$

5. Show that aggregate output and ex-ante welfare (i.e. welfare before you know your A and the sequence of ε) in this economy are increasing in σ_ε^2.Explain why

6. Show that aggregate output is constant in σ_A^2 and ex ante welfare is decreasing in σ_A^2. Explain why

7. Solve for the difference between labor income inequality and consumption inequality (measure inequality as the standard deviations of the log). What are the key parameters that determine this difference? Explain why

Question 4. Consider an economy with a continuum (measure 1) of ex-ante identical consumers, each living for two periods. The consumers have utility given by

$$\log(c_1) + \beta \log(c_2)$$

where c_1 and c_2 are period 1 and period 2 consumption and β is the discount factor. In period 1, each agent is endowed with y units of output which can be either consumed, c_1, or invested, k. In period two, consumers receive income from the capital they saved in period 1 and from inelastically supplying their labor endowment to the market. The labor endowment of any given individual is random and it is independent across agents. Period 2 labor endowments can be either $1 - \varepsilon$ or $1 + \varepsilon$, with $0 < \varepsilon < 1$; the probability that any agent's labor endowment is $1+\varepsilon$ is $1/2$. Due to the independence of shocks across consumers, a law of large numbers operates so that also the fraction of agents with labor endowment in period 2 equal to $1 + \varepsilon$ is $1/2$. That is, there is no uncertainty about the period-2 aggregate labor endowment: the supply of labor is constant at 1. In the second period, output is produced by perfectly competitive firms which operate a standard Cobb-Douglas production function: they sell the output to consumers and rent the capital and the labor services from the same consumers at rates r and w, respectively.

1. Write down the consumers problem and define a competitive equilibrium for this economy
2. For the case $\varepsilon = 0$ (no individual uncertainty) analytically solve for the competitive equilibrium. Argue that the equilibrium is efficient.
3. Show that the equilibrium interest rate is a decreasing function of ε.
4. For the case $\varepsilon > 0$ define the equilibrium risk faced by consumers as the ratio between equilibrium consumption in the high endowment state and equilibrium consumption in the low endowment state. Show that a) equilibrium risk *is not* affected by the equilibrium level of k and b) agents perceive that their individual risk *is* affected by their choice of k. Explain why this the case.
5. For the case $\varepsilon > 0$ show that the equilibrium capital is not efficient in the sense that ex-ante welfare of agents can be improved by changing the level of k?

For more on this issue see "Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks", by Davila, Hong, Krusell and Rios-Rull.

Question 5. Consider an economy with a continuum (measure 1) of ex-ante identical consumers each having efficiency units of labor ε which is i.i.d. over time and across agents, has non negative support and mean μ. Consumers can trade a non contingent bond a but cannot borrow ($a \geq 0$). Assume the optimal decision rule for assets has this form

$$a' = (1 + r)a + \gamma w\varepsilon$$

where $0 < \gamma < 1$ is a constant and r and w are equilibrium interest and wage rates. There are also competitive firms which hire labor and capital at prices w and r and operate a standard production function $AK^{\alpha}L^{1-\alpha} + (1-\delta)K$ where A is productivity and $1 > \delta > 0$ is the depreciation of capital

1. Show that a stationary distribution of consumers over assets and shocks exists only for $r < 0$.
2. For all $r < 0$ solve and plot the stationary supply of assets i.e. $E(a)$ and for demand for capital by the firms i.e. $K(r)$
3. Solve for equilibrium interest rate and wage rate.

4. Discuss what happens to long run equilibrium GDP, interest and wage rate if consumer want to save more (γ increases) or if productivity of the firms decline (A goes down) .