Macroeconomic Theory
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Macroeconomic Theory (8107)

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Problem set 2

Question 1. Distributions under complete and incomplete markets

1. Consider the following version of the economy discussed by Chatterjie. There are two consumers. Consumer 1 starts owning 20% of a representative firm, consumer 2 owns 80% of the firm. Assume that consumers can trade a full set of Arrow securities at time 0. The representative firm solves

$$\max_{\{k_j\}} \sum_{j=0}^{\infty} p_j \left(k_j^{\alpha} + (1-\delta) k_j - k_{j+1} \right)$$
$$\equiv \max_{\{k_j\}} \sum_{j=0}^{\infty} p_j d_j$$

where p_j is the price of consumption at date j relative to consumption at date 0. Assume that initial capital is 60% of the steady-state capital. Preferences of both consumer are given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t + \bar{c})$$

Assume that $\alpha = 0.4, \delta = 0.1, \beta = 0.9, \bar{c} = -0.1$. Solve and plot for the paths, as the economy transits from its initial condition to the steady state, of risk free interest rate, consumption and wealth of both consumers. Also plot the saving rate of both consumers where saving rate is $(s_{it}d_t - c_{it})/s_{it}d_t$

2. Now assume that consumers cannot trade any assets so in each period

$$c_{it} = s_{i0} d_t$$

(a) Assume that the firm chooses the same capital stock as in part 1. Solve and plot consumption and wealth for both consumers in this case. Would consumer 1 (the poor one) want the firm to choose a faster or slower growth of capital stock? How about consumer 2. Explain why and briefly discuss why, in general, when markets are not complete the problem of the firm is not uniquely defined

(b) Compute the price (in terms of proportional increase in lifetime consumption) that a consumer (before knowing its type) is willing to pay to live in the economy of part 1 rather than in the one of part 2.

Question 2. Optimal consumption with borrowing constraints

Consider an infinitely lived consumer with preferences given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

The consumer has 0 initial wealth ($a_o = 0$) and in each (discrete) time period she receives a constant endowment y. She faces a constant interest rate (at which she can borrow or lend) of r, so her budget constraint can be written as

$$c_t + a_{t+1} = a_t(1+r) + y$$

also assume that she faces a borrowing constraint of the form

$$a_{t+1} \geq -a$$

Assume the following parameter values: $y = 1, a = 4, r = 0.1$

- Solve and plot the paths for optimal consumption and asset accumulation for the following values of the discount factor β, $[0.5, 0.7, 0.99]$.
- In the last case ($\beta = 0.99$) verify that the transversality condition for optimality (see SLP theorem 4.15) holds at your proposed solution

Question 3. Optimal consumption with borrowing constraints and growth

Consider the following deterministic income fluctuation problem

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t.$$

$$\begin{aligned} c_t + a_{t+1} &= (1+r)\, a_t + y_t \\ a_{t+1} &\geq -\bar{a} \leq 0 \end{aligned}$$

with $u' > 0, u'' < 0$, $\beta(1+r) = 1$, $\bar{a} < \infty$ and $\{y_t\}_{t=0}^{\infty} = (1+g)^t$, $0 < g < r, a_0 = 0$

- Solve for the the optimal consumption and wealth path as a function of $\bar{a}$ (as warm-up start with the easy case in which $\bar{a} = 0$)

- Now assume that the borrowing constraint has the following form:

$$\frac{a_{t+1}}{y_{t+1}} \geq -\bar{a} < 0$$

Solve for the minimum value of $\bar{a}$ that allows the consumer to maintain a constant consumption for every period t.

Question 4. Precautionary saving and stock prices in partial and general equilibrium

Consider an economy which only lasts two periods. In the first period the representative consumer has non storable income $y = 1$ and in the second period she receives dividends from a tree. Dividends are equal to 1.1 with probability 0.5 and equal to 0.9 with probability 0.5. The consumer discounts the future at rate β.

- Assume the consumer has quadratic utility. Solve for the equity premium (i.e. the expected return on the tree minus the risk free rate) and the amount of saving (i.e. $y - c$) when the economy is closed and when the economy is open and can trade (in the first period) with the rest of the word a risk free bond that pays an exogenous rate of interest $(1 + r) = \frac{1}{\beta}$.

- Now assume that that the consumer has $CRRA$ utility, i.e. $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Plot equity premium and saving for the closed and the open economy for γ going from 1 to 10.

- Repeat the exercise above for exponential utility i.e. $u(c) = -e^{-\gamma c}$

Write a paragraph explaining why the response of the equity premium to utility curvature is different in the open and in the closed economy.