Macroeconomic Theory
Fabrizio Perri

Macroeconomic Theory (8107)

Spring 2012, Mini 1

Problem set 1

Due Tuesday January 31, in class

1. Problems 4.B.1 and 4.B.2 Mas-Colell, Whinston and Green

2. Consider a static economy composed by I consumers each having the following utility function
$$U(c, l) = c - \frac{l^{\phi}}{\phi}, \phi > 1$$
where c is consumption and l is hours worked. Assume that each consumer supplies labor with different levels of efficiency e_i so its budget constraint reads
$$we_i l_i = c_i$$
where w is the (exogenously given) wage rate per efficiency unit of labor.

   (a) Show that in general aggregate labor supply and aggregate consumption demand depend on the distribution of skills across consumers

   (b) Explain why this is the case even if preferences of consumers are quasilinear in consumption.

3. Problem 8.1 Sargent Ljunqvist

4. Consider a closed economy with two consumers (indexed by i) which live for T periods. Each consumer receives an endowment of perishable income in each period. Endowments are given by the deterministic sequences $\{y_{i,t}\}_{t=1,T}$. Consumers have preferences given by
$$\sum_{t=1}^{T} \beta_i^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

where $\beta_1 \neq \beta_2$. and they can trade a risk free bond. Show that there exist a representative agent, i.e. a fictitious single agent economy, the single agent has endowment equal to $\{Y_t = y_{1,t} + y_{2,t}\}_{t=1,T}$, preferences are given by

$$\sum_{t=1}^{T} \beta^t \frac{c_t{}^{1-\sigma}}{1-\sigma}$$

and whose first order conditions can be used to solve for equilibrium interest rates in the original 2 agent economy. Solve for the discount factor of the RA, i.e. β as a function of the preference parameters of the two consumers

5. Consider a discrete time closed economy with two infinitely lived consumers and two possible states of the word. In state 1 (boom) the total endowment is $1+\varepsilon$, in state 2 (recession) the total endowment is $1-\varepsilon, 0 < \varepsilon < 1$. In each period the probability of the high state is p. Consumer 1 receives a constant share γ of the total endowment and the remaining goes to consumer 2.

   (a) Choose p so that, conditional on being in a boom, the average duration of a boom is 4 periods (Recall that expected duration of a boom conditional of being in a boom is given by

   $$\sum_{j=1}^{\infty} jp^j$$

   Conditional on being in a recession, how long you expect a recession to last?

   (b) Consider the case in which the two consumers have identical preferences given by

   $$E\sum_{t=0}^{\infty}\sum_{s^t} \beta^t \frac{c(s^t)^{1-\sigma}}{1-\sigma}$$

   and they can trade a full set of assets contingent on all realizations of s^t, i.e. all possible sequences of booms and recessions. Solve for the equilibrium allocation and argue that asset prices do not depend on γ.

   (c) Assume $\sigma = 3$ and $\varepsilon = 0.02$. Plot the average real risk free rate in the economy, i.e. the return on an asset that pays one unit of consumption for sure next period as a function of β. Pick β so to match a risk free rate of 1%.For this β compute the price of stocks (a stock is an asset which pays off a dividend equal to the aggregate endowment every period) in booms and recessions (you should be able to compute this almost analytically). Compute the average equity premium i.e. the difference between the average returns on stock and the average returns on bond. Repeat the exercise using $\varepsilon = 0.03$ (with $\sigma = 5$) and $\sigma = 10$.(with $\varepsilon = 0.02$). How is the equity premium affected by aggregate risk (ε) and by risk aversion (σ).

(d) Now assume that consumers have heterogenous preferences and that consumer 1 has a risk aversion of 5, while consumer 2 has risk aversion 2, but still assume that consumers can trade a full set of contingent assets. Compute equilibrium allocations (using the Negishi algorithm) for $\gamma = 0.3$ and for $\gamma = 0.9$. Compute average equity premia in the two economies (use the same β you computed in point c). Compare this equity premium with the one in which both consumers have risk aversion equal to 5. Briefly comment your result. What happens to the equity premium when you increase the wealth of the more risk averse agent? Explain why? How would you answer change if agent 2 were risk neutral?

6. Consider a representative agent which owns a tree yielding a non storable fruit d_t in every period. The fruit of the tree is the only endowment of the economy. In each period the dividends are uniformly distributed between the values a and b , $b > a > 0$. Preferences of the representative agent are given by

$$E \sum_{t=0}^{\infty} \beta^t \log(c_t), 0 < \beta < 1$$

(a) Solve for the consumption equilibrium allocation

(b) Solve for the price of the tree in period t as a function of the dividend realization d_t

(c) Solve for the risk free rate. Is the risk free rate pro-cyclical or counter-cyclical? Explain why

(d) Consider a range of options indexed by the parameter $\bar{p} \in [a\frac{\beta}{1-\beta}, b\frac{\beta}{1-\beta}]$. An option is a claim that entitles its holder to buy, if she wants so, a tree next period at a price $\bar{p}$. Solve for the price of an option $q(\bar{p})$ and plot it as a function of all possible $\bar{p}$

7. Consider the same economy as above but now the process for d_t is *deterministic* and alternates three states. In state 1 (beginning of boom) the fruit will grow at rate ε over each of the next two periods. In state 2 (mid boom) the fruit will grow at rate ε next period but shrink at rate ε the period after and in state 3 (recession) the fruit will shrink at rate ε over the next period and grow at rate ε the period after. So the process can be written as

Period	0	1	2	3	4	5	6	..
State	1	2	3	1	2	3	1	..
Dividend	d_0	$d_1 = d_0 e^{\varepsilon}$	$d_2 = d_1 e^{\varepsilon}$	$d_3 = d_2 e^{-\varepsilon}$	$d_4 = d_3 e^{\varepsilon}$	$d_5 = d_4 e^{\varepsilon}$	$d_6 = d_5 e^{-\varepsilon}$	..

$\varepsilon > 0$

(a) Plot a typical sample path for GDP and equilibrium consumption over time in this economy.

(b) Solve for one period risk free rates in the economy in each state and plot them over time. What happens to interest rates when the economy moves from midboom into a recession and when the economy moves from recession to beginning boom? Explain why.

(c) Solve for the (annualized) rate for a two-periods risk free bond (i.e. a bond which pays 1 unit for sure in 2 periods) in each state.

(d) Plot the differences between the annualized rate for the two period bond and the return on one period bond in the each state. This is a measure of the slope of the yield curve (i.e. the curve that plots returns on bonds of different maturities against the maturity structure) . In the data in periods preceding recessions the yield curve tend to be "inverted", i.e. return on long bonds are lower than returns on short bonds. Is this model consistent with this fact? Explain why.

8. Consider the following two periods economy. Preferences of the representative agent are given by

$$\begin{aligned} \frac{c_1^{1-\sigma}}{1-\sigma} + \beta\left(\frac{c_2^{1-\sigma}}{1-\sigma} - \frac{l_2^{1+\gamma}}{1+\gamma}\right), 0 &< \beta < 1 \\ \sigma &> 0, \gamma \geq 0 \end{aligned}$$

where c is consumption, and l is labor. The representative agents owns measure 1 trees which cannot be eaten, do not depreciate from period 1 to period 2 and fully depreciate after period 2. The trees yield a non-storable dividend of 1 in the first period and in the second period they are rented out to competitive firms that combine them with labor to produce output according to the technology

$$y = e^z k^\alpha l^{1-a}$$

where z is a normal disturbance with mean 0 and variance η, k is the measure of trees rented by the firms and l is labor rented by firms. The economy is closed but agents can trade a real risk free bond.

(a) Write down budget constraint of the agents, define a competitive equilibrium and solve for it.

(b) Solve for the risk free rate, for the price of the tree (after dividend in period 1 has been paid) and for the equity premium (i.e. expected return on equity minus the risk free rate).

(c) Fix $\sigma = 2$ and show how the equity premium changes as you vary γ from 0 to 5. Give some economic intuition for your result. (For more on this see, "Risk Aversion and the Labor Margin in Dynamic Equilibrium Models", Eric Swanson, AER, 2012)