University of Minnesota

8107 Macroeconomic Theory, Spring 2012, Mini 1

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1 Limited enforcement of contracts

So far we have analyzed environments in which allocations are not efficient due to the presence of incomplete markets. In this class we are considering another possible reason for which allocations are not efficient, i.e. the imperfect enforcement of intertemporal contracts. In a Arrow-Debreu world enforcement is not an issue and this means that implicitly we assume that punishment for not honoring an intertemporal contract is infinity. Here we abandon this hypothesis and we assume that agents can default on intertemporal promises and if they so they suffer *some* punishment. Punishment is usually modeled as (temporary or permanent) exclusion from future credit market participation. We will do two things in the lecture: first we will characterize the set of constrained efficient allocations in a simple two agents economy in which punishment or enforcement is limited. We'll then briefly discuss how to characterize allocations with limited enforcement in an economy with a continuum of agents and how these allocations are different from the ones with incomplete markets as in Aiyagari or Huggett.

1.1 A 2 Agent economy

Consider an economy with 2 (types of) infinitely lived households, $i = 1, 2$,who each period receive symmetric stochastic endowments $e_t^i = e_t^i(s_t) > 0$ where s^t denote the state of the world at date t. Households preferences are standard and given by:

$$U(c) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi(s^t) u(c_t^i(s^t))$$

In this economy we can define

$$V^{i,Aut}(s^t) = \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau-t} \pi(s^\tau | s^t) u(e_\tau^i(s^\tau))$$

as the minimum continuation utility that each agent can guarantee for herself. Note that if $e_\tau^i(s^\tau)$ can take the value of 0 and utility satisfies the Inada conditions the $V^{i,Aut}(s_t) = -\infty$. Now consider the standard planning problem i.e.

$$\max_c \lambda U(c^1) + (1-\lambda) U(c^2)$$

s.t.

$$c_t^1(s^t) + c_t^2(s^t) \quad \leq \quad e_t^1(s^t) + e_t^2(s^t) \text{ for each } s^t$$

where λ are the Pareto weights and define continuation lifetime expected utility from allocation (c^1, c^2) for agent i in node s^t

$$\begin{aligned} U(c^i, s^t, \lambda) &= u(c^i_t(s^t)) + \\ & \sum_{\tau=t+1}^{\infty} \sum_{s^\tau | s^t} \beta^{\tau - t} \pi(s^\tau | s^t) u(c^i_\tau(s^\tau)) \end{aligned}$$

Now consider an allocation solution to the planning problem for which there exist a state s^t in which $U(c^i, s^t, \lambda) < V^{i,Aut}(s^t)$. If the planner tried to implement that allocation at that node s^t agent i would opt to exit the economy and live in autarky so that allocation cannot be a solution to the planning problem. In other words limited enforcement of contracts limits the allocations that can be implemented by a planner. So constrained efficient-allocations in a world with limited enforcement solve the following problem

$$\begin{aligned} & \max_c \lambda U(c^1) + (1 - \lambda) U(c^2) \\ & \text{s.t.} \\ c^1_t(s^t) + c^2_t(s^t) &\leq e^1_t(s^t) + e^2_t(s^t) \text{ for each } s^t \\ U(c^i, s^t) &\geq V^{i,Aut}(s^t) \text{ for each } i, s^t \end{aligned}$$

Computing and characterizing these allocations is in general harder than computing the simple planning solution as consumption in a given state does not only enter the objective function and the resource constraint but also all the enforcement constraints that precede that state. In this note we'll work a simple case in which these are easy to compute. Consider the case in which the state of the world $s_t \in S = \{1, 2\}$ and endowments are given by

$$\begin{aligned} e^1(s_t) &= \begin{cases} 1 + \varepsilon & \text{if } s_t = 1 \\ 1 - \varepsilon & \text{if } s_t = 2 \end{cases} \\ e^2(s_t) &= \begin{cases} 1 - \varepsilon & \text{if } s_t = 1 \\ 1 + \varepsilon & \text{if } s_t = 2 \end{cases} \end{aligned}$$

States change according to the transition probabilities

$$\pi = \begin{pmatrix} \delta & 1 - \delta \\ 1 - \delta & \delta \end{pmatrix}$$

Obviously in this world the symmetric first best allocation is the one in which each agent consume exactly 1 in each state and so we can define the continuation value from the first best allocation as $U^{FB} = \frac{u(1)}{1-\beta}$ for all s^t. Solving for the value of autarky amount in solving the following two equations and two unknowns

$U(1+\varepsilon)$ and $U(1-\varepsilon)$

$$\begin{aligned} U(1+\varepsilon) &= u(1+\varepsilon) + \beta\left[\delta U(1+\varepsilon) + (1-\delta)U(1-\varepsilon)\right] \\ U(1-\varepsilon) &= u(1-\varepsilon) + \beta\left[\delta U(1-\varepsilon) + (1-\delta)U(1+\varepsilon)\right] \end{aligned}$$

and yields

$$U(1+\varepsilon) = \frac{1}{D}\left\{u(1+\varepsilon) + \frac{\beta}{1-\beta}(1-\delta)\left[u(1+\varepsilon)+u(1-\varepsilon)\right]\right\}$$

$$U(1-\varepsilon) = \frac{1}{D}\left\{u(1-\varepsilon) + \frac{\beta}{1-\beta}(1-\delta)\left[u(1-\varepsilon)+u(1+\varepsilon)\right]\right\}$$

where

$$D = \frac{(1-\beta\delta)^2 - (\beta-\beta\delta)^2}{1-\beta} > 0$$

Once we characterize these three values we can prove the following (see Kehoe and Levine, 2001 for more details)

Proposition 1 *Constrained-efficient consumption distribution is characterized by a number $\varepsilon_c(\varepsilon) \geq 0$. The agent with income $1+\varepsilon$ consumes $1+\varepsilon_c(\varepsilon)$ and the agent with income $1-\varepsilon$ consumes $1-\varepsilon_c(\varepsilon)$ regardless of her past history. The number $\varepsilon_c(\varepsilon)$ is the smallest non-negative solution of the following equation*

$$\max(U^{FB}, U(1+\varepsilon)) = U(1+\varepsilon_c(\varepsilon))$$

The figure below gives a graphical exposition of the proof and we'll discuss it in class. Notice that here the size of individual risk, captured by the parameter ε,affects how far the constrained efficient allocation is from the unconstrained efficient: if risk is very large then the value of autarky in both states lie below the first best and first best is implementable by the planner: the value of autarky is so low that no agent would ever want to take it. If risk is very small on the other hand then autarky is an attractive option for the agent with high income so no risk sharing can be obtained. If risk is in the middle region between ε_1 and ε_2 only partial risk sharing is obtained, i.e. the rich agent transfers some of the consumption to the poor agent but equal consumption is not incentive compatible.

Now that we know how to compute allocations we might wonder whether these allocations can be decentralized as equilibria. There are three ways of doing this. One is to show that these allocations emerge as subgame perfect equilibrium of transfer game (Kocherlakota 1996) or as competitive equilibrium with enforcement constraints (Kehoe and Levine 1993) or finally as sequential markets equilibrium with state-contingent borrowing constraints (Alvarez and Jermann 2000).

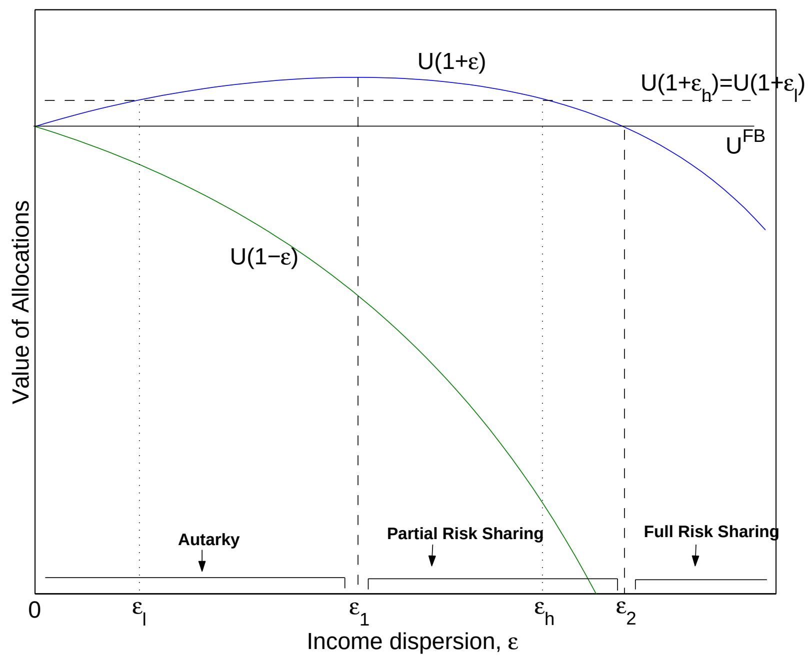


Figure 1: Constrained efficient allocations

2 A Continuum of agents Economy

Here we just briefly sketch a methodology that is useful to characterize efficient allocations in economies with a continuum of agents. Assume that there measure 1 of infinitely lived agents each facing an idiosyncratic income process $\{y_t\}$ with invariant distribution Π (just as in a standard Aiyagari economy). The key difference here is that enforcement of contracts is limited and agents have the option of going to autarky. Atkeson and Lucas (1992, 1995) have proposed a method for characterizing efficient allocations that consist in having an many individual planners (i.e. a planners which chooses allocation for a given agent) minimizing the cost of delivering a given level of promised utility to a particular agent; these planners then can inter-temporally trade resources among themselves at a rate R and the equilibrium interest rate R is the one which guarantees resource feasibility. Sometimes this method is referred to as a "component planning problem". This method works well to characterize efficient allocations in economies with limited enforcement but it was first used to characterize efficient allocations in economies with private information.

2.0.1 Recursive Component Planning Problem

We first define the relative "price" of resources today versus tomorrow $1 + r = R \in (1, \frac{1}{\beta})$ and then define the function $C \equiv u^{-1}$ which is the inverse of the utility function and gives the amount of current consumption needed to generate period utility u. Each social planner allocate utilities instead of consumption and use promises of expected discounted utility w and income y as state variables and current utility h and expected utility from tomorrow onward, conditional on tomorrow's income shock y', $g(y')$ as control variables. The Bellmann equation can then be written as

$$\begin{aligned} V(w,y) &= \min_{h,g(y')} \left(1-\frac{1}{R}\right) C(h) \\ &\quad + \frac{1}{R} \sum_{y' \in Y} \pi(y'|y) V(g(y'),y') \\ w &= (1-\beta)h + \beta \sum_{y' \in Y} \pi(y'|y) g(y') \\ g(y') &\geq U^{Aut}(y') \end{aligned}$$

Note that here $V(w,y)$ is the total resource cost the component planner has to minimally spend in order to fulfill utility promises w, without violating the individual rationality constraint of the agent. Note also that if the income process is *iid*, y is no longer a state variable. Once we solve the planning problem for a given individual (and for a given interest rate R) we can, using transition function just as we did in the Ayiagari economy, find a stationary distribution over utility promises and income shocks; we denote this stationary distribution as Ψ_R. In order to do so we first define the transition function Q_R induced by π and $g(w,y;y')$ as

$$Q_R((w,y),(A,\mathcal{Y})) = \sum_{y' \in \mathcal{Y}} \begin{cases} \pi(y'|y) & \text{if } g(w,y;y') \in A \\ 0 & \text{else} \end{cases}$$

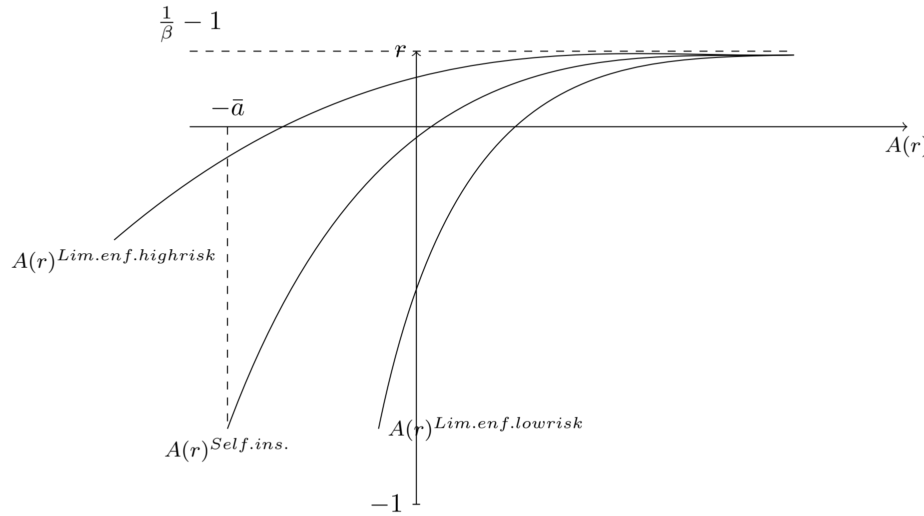
and the define the stationary distribution Ψ_R over (w,y) as

$$\Psi_R(A,\mathcal{Y}) = \int Q_R((w,y),(A,\mathcal{Y}))d\Psi_R$$

Of course one has to prove that invariant measure exists and that is unique (proofs for the i.i.d. case are contained in Atkeson and Lucas, 1995) and if this the case one can define the aggregate saving demand function exactly as in the Ayiagari economy

$$A(1+r) = \int y d\Psi_R - \int C(h(w,y))d\Psi_R$$

One key difference between this function and the one obtained in the self insurance economy is that the one in the self insurance economy depended on exogenously specified borrowing constraint while in this case constraints on the state space are endogenous and depend on the value of autarky. As in the 2 agents case, one can show that the allocations that solve this "dual" component planning problem are also equilibrium allocation in a competitive setting in which agents can trade a full set of Arrow securities but they are subject to a limit in the amount in which they go short on each security. Whether the aggregate



saving demand function in this limited enforcement economy is to the right or to the left of the aggregate saving demand function of the self-insurance economy will in general depend on the fundamentals of the economy. If, for example, agents face very little individual risk then in the limited enforcement economy little borrowing is sustainable so the saving function in the limited enforcement will lie to the right of the one in the self insurance. If on the other hand individual risk is high, the stationary allocation in the limited enforcement will display a lot of risk sharing and since high level of borrowing are sustainable agents will tend to decumulate wealth and $A(r)$ will lie to the left of the one in the self insurance economy.