University of Minnesota
8107 Macroeconomic Theory, Spring 2012, Mini 1
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1 Lecture 2. Asset pricing with the representative agent

In this class we will analyze merits and limitations of a commonly used application of application of model economies in which the RA holds, that is asset pricing. The key idea is that aggregate consumption here is a sufficient statistic to price assets. Consider again the basic economy we discussed in the previous lecture i.e. many consumers with identical preferences and endowment of non storable goods. In this economy it is easy to solve for equilibrium allocation using the planning problem. Once the allocation are computed assets can be priced, even though in equilibrium they are not traded (for more details on this see Sargent, 1987, section 3.5. Here we will first discuss some examples and then discuss one famous issue in asset pricing, i.e. the equity premium puzzle.

2 Examples of asset pricing

2.1 Pricing a one period risk free bond

$$q^1 = \beta E \frac{u'(c')}{u'(c)}$$

Now assume that period utility is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

then the pricing becomes

$$q^1 = \beta E \left(\frac{c'}{c}\right)^{-\gamma}$$

or writing everything in terms of risk free interest rate R^f

$$R^f = \frac{1}{q^1} = \frac{1}{\beta} \frac{1}{E g^{-\gamma}} \tag{1}$$

where $g = \frac{c'}{c}$. This pricing equation tells you how interest rate should move with change in expected growth in consumption. As expected growth increases agents would like to borrow to increase their current consumption, but since that is not an equilibrium, the equilibrium real interest rate, has to increase to clear credit markets. Notice that in this case γ, which is the inverse of intertemporal elasticity of substitution (IES), is the elasticity of interest rate to changes in expected growth. If γ is high then small changes in expected growth will cause large movements in equilibrium interest rates. It is common in asset pricing to define

$$m_{t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

where the term m_{t+1} is referred to as the pricing kernel or the stochastic discount factor. Note that using this notation we have

$$\frac{1}{E(m_{t+1})} = R_f \tag{2}$$

2.2 Multi-period bonds

Consider pricing a bond that pays a fixed coupon of 1 for 2 periods. Such a bond, one period before maturity, is just the bond analyzed in the previous case hence the pricing equation can be written as

$$\begin{aligned} q^2 u'(c) &= \beta E u'(c')(1 + q^{1\prime}) \\ q^2 &= \beta E \frac{u'(c')}{u'(c)} + \beta^2 E \frac{u'(c'')}{u'(c)} \end{aligned}$$

2.3 Risky assets

Consider pricing an asset that entitles the holder to future contingent pay-outs, as for example a defaultable bond, an option or any financial derivative:

$$q_t = \beta E \frac{u'(c_{t+1})}{u'(c_t)} d_{t+1}$$

then the condition can be written as

$$\begin{aligned} q_t &= E m_{t+1} d_{t+1} \\ &= E(d_{t+1})E(m_{t+1}) + cov(m_{t+1}, d_{t+1}) \\ &= \frac{E(d_{t+1})}{R_f} + cov(m_{t+1}, d_{t+1}) \end{aligned} \tag{3}$$

where the last equality uses equation (2).

Equation (3) reveals the two fundamental components of asset pricing, which are the expected discounted value of the pay-outs (captured by the first term) and the covariation of the pay-outs with marginal utility (captured by the second term), which detrmines the so-called risk premium of the asset. Notice for example that if agents are risk neutral then $m_{t+1} = \beta$ and only the first component appears and

$$q_t = \frac{E(d_{t+1})}{R_f}$$

is the price of an assets is simply the discounted expected value of the asset payout.

2.4 Pricing claims to the stock market

The stock market as a whole can be interpreted as a claim to share of aggregate consumption, i.e. $d_t = c_t$ So the pricing equation in this case is

$$p_t = \beta E \left(u'(c_{t+1}) \frac{u'(c_{t+1})}{u'(c_t)} (d_{t+1} + p_{t+1}) \right)$$

doing repeated substitutions and taking limits yield

$$p_t = \sum_{j=1}^{\infty} \beta^j d_{t+j} \frac{u'(c_{t+j})}{u'(c_t)} = \sum_{j=1}^{\infty} \beta^j c_{t+j} \frac{u'(c_{t+j})}{u'(c_t)}$$

It is instructive to notice that in the the special case of log utility this reduces to

$$p_t = \frac{1}{u'(c_t)} \sum_{j=1}^{\infty} \beta^j = \frac{\beta}{1-\beta} c_t = \frac{\beta}{1-\beta} d_t$$

where the price depends on current consumption but does not depend on future consumption nor on expected consumption growth. Why?

Notice that in general the pricing depends on 3 elements: preferences, allocations and dividends streams of the assets. In the case of time separable CRRA, preferences are summarized by 2 parameters, one is the discount factor β the other is the curvature parameter (which is at the same time risk aversion and the reciprocal of the intertemporal elasticity of substitution). If we want to make more progress on solving for prices and returns we can assume that log dividends are a random walk

$$\begin{aligned} d_{t+1} &= d_t e^{\varepsilon_t} \\ \varepsilon_t &\rightarrow N(\mu, \sigma^2), i.i.d. \end{aligned}$$

This yields expressions for the risk free rate R^f and for the expected return on stocks $E(R^s)$

$$\begin{aligned} R^f &= \frac{1}{E(\beta \frac{d_t}{d_{t+1}})} = \frac{1}{\beta e^{-\mu + \frac{1}{2}\sigma^2}} = \frac{1}{\beta} e^{\mu - \frac{1}{2}\sigma^2} \\ E(R^s) &= E(\frac{d_{t+1} + p_{t+1}}{p_t}) = \frac{1}{\beta} E \frac{d_{t+1}}{d_t} = \frac{1}{\beta} E\varepsilon_{t+1} = \frac{1}{\beta} e^{\mu + \frac{1}{2}\sigma^2} \end{aligned}$$

Note how the variance of dividends (and of consumption) lowers returns on bond and increases return on stocks. Bond return falls with σ^2 as bonds, being riskless, are more attractive with higher variance. Since bonds are in zero supply (i.e. nobody can buy or sell bonds) an increase in demand implies that their equilibrium price raises and their return falls. Stocks on the other hand are less attractive with higher risk, therefore, by the same logic, their price fall and their expected return increase.

3 The equity premium puzzle

Rajnish Mehra and Ed Prescott in a paper that appeared in 1985 of the Journal of Monetary Economics pointed to a puzzle i.e. to some pricing implications of the representative agent model that did not square well with observed prices. Assume that a period is a quarter. Let R^s be the annualized return on stocks and R^f be the return on bonds over a quarter. Assume that preferences are time separable CRRA. In this case we can define the stochastic discount factor m_t as

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}$$

and it is easy to derive the following restriction implied by the asset pricing equations discussed above

$$E_t(m_{t+1}(R^s - R^f)) = 0 \tag{4}$$

now using the definition of covariance we can rewrite (4) as

$$E(R^s - R^f)E(m_{t+1}) = -cov(m_{t+1}(R^s - R^f))$$

or

$$\frac{E(R^s - R^f)}{\sigma_{R^s - R^f}} = -corr(m_{t+1}(R^s - R^f))\frac{\sigma_{m_{t+1}}}{E(m_{t+1})} \tag{5}$$

The left hand side of (5) can be measured directly in the data and it is a common measure of the ratio of returns to risk in asset markets called the Sharpe ratio. In postwar US data, the mean annualized return of stocks over bonds is about 8% with a standard deviation of about 16%, so the Sharpe ratio is about 0.5. Also assume that $\frac{c_{t+1}}{c_t}$ is log-normal i.e. that $\log c_{t+1} - \log c_t$ is normal with mean μ_c and standard deviation σ_c, so that m_t is also log normal with mean $-\gamma\mu_c$ and standard deviation $\gamma\sigma_c$. Notice that the expression $\frac{\sigma_{m_{t+1}}}{E(m_{t+1})}$ is also called the coefficient of variation which in the case of a log normal distribution is equal to $\sqrt{e^{\sigma} - 1} \simeq \sigma$ so that

$$\frac{E(R^s - R^f)}{\sigma_{R^s - R^f}} \simeq -corr(m_{t+1}(R^s - R^f))\gamma\sigma_c$$

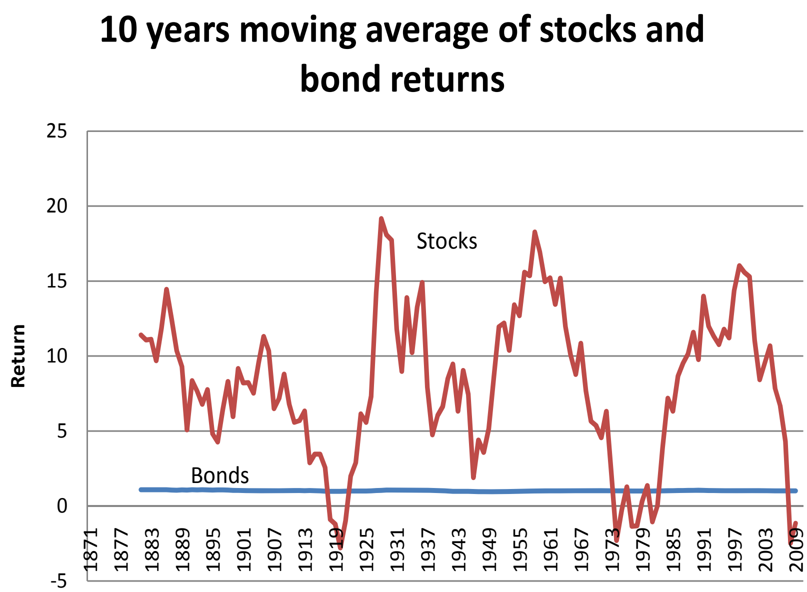
now since $\left|corr(m_{t+1}(R^s - R^f))\right| \leq 1$ we have that

$$\frac{\left|E(R^s - R^f)\right|}{\sigma_{R^s - R^f}} \leq \gamma\sigma_c \tag{6}$$

Now simply use post war US data to estimate $\sigma_c \simeq 1\%$ which impies that γ must be at least 50 (Notice that value would be even larger if $corr(m_{t+1}(R^s - R^f))$ is less than 1 in absolute value). Inequality 6 is a special case of what in finance is usually referred to as Hansen Jagannathan bound. The intuition behind this result is that in order for stock to command a very high premium over bonds it must be that the representative agent really dislike stocks and in this model the only reason why he or she dislikes them is because they pay low returns in state in which its marginal utility is high i.e. $corr(m_{t+1}(R^s - R^f)) < 0$ and so they create additional risk and the agent really hates risk (i.e. high risk aversion). One problem with such a high value for γ his is a value that is hard to justify based on studies of individual preferences. But even if you are willing to accept high risk aversion there is an additional problem with picking a very high value for γ, that it also implies very low value for the intertemporal elasticity of substitution. If you go back to equation 1 you see that high value of γ implies extremely high values for R^f because if agents are really unwilling to substitute then it takes an extremely high interest rate to deter them from borrowing in anticipation of future growth. That again can be solved if you are willing to assume a very low value for β, but still a high γ would imply that the interest rate should be extremely sensitive to changes in growth expectations, which we do not really see in the data.

3.1 Explanations for the equity premium puzzle

The equity premium puzzle has generated an enormous literature trying to explain it which would be impossible to summarize here. Three very good surveys are an article by Kocherlakota, a survey article by Mehra and a more recent one by Cochrane (both posted on the webpage). At the very basic level the EP puzzle is the following question: why stocks pay such a high return if they are not that risky? (see the figure below)



An overly simplified classification of the explanations for the equity premium puzzle would break them down in three types: preferences, endowments and non representative consumer.

3.1.1 Preference based explanations

The simplest preference-based explanation is the high risk aversion, which, as we have seen above, has problems. Other preference explanation include

- Non time separable preferences which can separately control risk aversion from the IES (You'll see more about this in the review session).
- Habits, i.e preferences in which my utility does not depend only on my own consumption but on the difference between my consumption and average consumption (these are called external habits i.e. catching up with the Jones), or on differences between current consumption and past values of my own consumption (internal habits). For the most sophisticated version of habits see a paper by Campbell and Cochrane, (1999) which reverse engineers the type of preferences that are needed to explain many features of several asset prices. Habits in general make agent more risk averse with implying a extremely low IES.
- Ambiguity aversion preferences (see Hansen, Sargent and Tallarini, 1999) which makes agent dislike stocks more, as agents have distorted (in the Murphy's sense) beliefs.

3.1.2 Endowment based explanations

The endowment explanations either resort to so called ”disasters” i.e. to the idea that stocks are subject to a rare but catastrophic risk (Rietz, 1988 and a series of recent papers starting with Barro, 2006), or to the so called ”Long run risk” i.e. the idea that stocks contain a small component of a very persistent (more than random walk) process which is hard to detect from data (for econometricians) but investors recognize, and so the premium is compensation for this long run risk (Bansal and Yaron, 2005). A recent paper by Van Binsbergen et al. (2011) though questions the validity of this explanation, by constructing a synthetic short run stock i.e. a one period claim to dividends. They then show how this assets also commands a premium (pretty much like stocks), suggesting that the the bulk of the equity premium is not compensation for long run risk.

3.1.3 Non representative agent explanations

These explanations are based on models that abandon the RA construct and suggest that the MRS used to price stock should not be the MRS based of the RA (i.e. based on aggregate consumption) but rather the MRS of a subset of consumers. For example consider the following assumption: Only 10% of the US households are allowed to hold stock (this can justified assuming a fixed cost of buying stocks). Under this assumption it is possible to show that there is an equilibrium in which the 10% hold all the US stock (worth say 1 GDP) while the remaining 90% hold a positive net position in bonds (say 50% of GDP). Notice that in this equilibrium stocks are more risky as, for example, a 1% decline in stock value implies a 2% decline in wealth of the stock holders (the stock position of the stock holders is leveraged). This higher risk could be an explanation for the higher return we see in the data. For more on this see Guvenen, 2010. One interesting issue is whether individual, uninsurable labor risk can generate equity premium. Intuitively it would seem that when agents are subject to a large idiosyncratic risk (i.e they could not trade Arrow securities contingent on the the realization of their labor income, they should be less willing to take additional risk (i.e. less willing to buy stocks). It turns out that this reasoning is partly incorrect as, if aggregate risk (i.e. stock market risk) and idiosyncratic risk (i.e. labor income risk) are not correlated and if preferences are identical and homothetic, the equity premium is not affected by the presence of idiosyncratic risk. In other words a special case of the aggregation result hold: the equity premium that prevails in an economy with incomplete markets is the same as the one that prevails in the economy with a representative agent. For more on this see Krueger and Lustig (2009) and try to solve the past prelim question below.

Pricing aggregate risk with idiosyncratic shocks

Consider a two-periods, one-good economy inhabited by a continuum of measure 1 of agents with identical preferences given by

$$\log c_1 + \log c_2$$

In period 1 all agents are identical as they each receive a perishable endowment of 1 and own equal shares s_1 in a single Lucas tree which yields stochastic dividends z in period 2. Dividends z are equal to z_h with probability $0 < \pi < 1$ and to z_l with probability $1 - \pi$, with $z_h > z_l > 0$. A fraction α of the fruit of the tree goes to the owners of the tree in period 2 and a fraction $1 - \alpha$ goes to workers, in proportion to their stochastic (unknown in period 1) idiosyncratic labor endowment in the second period, denoted by η_{2i}, so that agent i labor income in the second period is given by $\eta_{2i}(1 - \alpha)z$.
For all i, η_{2i} is equal to η_h with probability $0 < \mu < 1$, and to η_l with probability $1 - \mu$, with $\eta_h > \eta_l > 0$, $E_1(\eta_{2i}) = 1$ and the realization of η_{2i} is independent on the realization of z. Assume also that a law of large numbers apply so $\int_i \eta_{2i} = 1$.

1. Assume that in period 1 agents can trade shares in the Lucas tree and Arrow securities contingent on the realization of their idiosyncratic labor endowment shock.

   (a) Write down period 1 and period 2 budget constraints and define a competitive equilibrium

   (b) Solve for individual consumption in equilibrium (Hint: it is easier to solve for equilibrium allocation using an appropriate planning problem and invoking the relevant welfare theorems)

   (c) Solve for period 1 prices of a risk free bond and of the Lucas tree

2. Assume now that the only asset traded are shares in the Lucas tree and a risk free bond

   (a) Write down period 1 and period 2 budget constraints and define a competitive equilibrium

   (b) Show that there is an equilibrium where no assets are traded and solve for equilibrium consumption in both periods

   (c) Solve for period 1 prices of a risk free bond and of the Lucas tree

   (d) Show that the price of the risk free bond is higher in this economy that in the economy in point 1. Why?

   (e) Show that ratio between the price of the risk free bond and the price of the Lucas tree is the same in this economy and in the economy in point 1. What does this suggest for the equity premium in these two economies?

3.2 Suggested Readings

1. Bansal, R. and A. Yaron (2005) "Risks for the long run: A Potential Resolution of Asset Pricing Puzzles", Journal of Finance

2. Barro (2006) "Rare disasters and asset markets in the twentieth century", Quarterly Journal of Economics

3. Van Binsbergen, J., Brandt, M. and R. Koijen, (2011) "On the Timing and Pricing of Dividends", American Economic Review

4. Campbell, J and J Cochrane. (1999) "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior.' Journal of Political Economy

5. Hansen, L. , T. Sargent and T. Tallarini (1999) "Robust Permanent Income and Pricing", The Review of Economic Studies

6. Guvenen F. (2009) "A Parsimonious Macroeconomic Model for Asset Pricing", Econometrica

7. Kocherlakota N. (1996) "The equity premium: it is still a puzzle", Journal of Economic Literature

8. Krueger, D. and H. Lustig (2009) "When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)?" Journal of Economic Theory

9. Rietz, T (1988) "The Equity Risk Premium: A Solution.' Journal of Monetary Economics

10. Sargent, T. (1987) "Dynamic macroeconomic Theory"