Long run growth 3: Sources of growth

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In the previous lecture we concluded that the return to capital that is $MPK + 1 - \delta$, is a key factor behind saving decisions. This class we will analyze what determines returns to capital. The marginal product of capital is how much additional output gets produced with an additional unit of capital. Technically it is the derivative of output relative to the capital stock, but with the production function we are using it turns out to be simply proportional to the average productivity of capital, where the factor proportionality is given by α,that is

$$MPK = \alpha \frac{Y}{K}$$

If we write the production function

$$Y = AK^{\alpha}(L * H)^{1-\alpha}$$

where K is capital stock, L is labor input, H is the average level of education of the workforce and A is total factor productivity (the efficiency of production) (Relative to the case analyzed in the previous class we not considering raw labor input but we are weighting it by the education) we obtain that

$$MPK = \alpha \frac{AK^{\alpha}(L * H)^{1-\alpha}}{K} = \alpha A H^{1-\alpha} (\frac{L}{K})^{1-\alpha} = \alpha A \frac{H^{1-\alpha}}{k^{1-\alpha}} \tag{1}$$

that shows the key determinants of the marginal product of capital. The marginal product of capital depends positively on α (the share of capital in production), on the average education of the workforce, on the total factor productivity A and depends negatively on the level of capital stock per worker (due to decreasing returns). The evidence suggests that α does not vary greatly across countries, so if we want to understand why returns to capital differ across countries we should focus on A, H and k. In figure 1 we see how different levels of total factor productivity are key in determining the performance of an economy. Notice also that high returns to capital are necessary and sufficient condition for factor accumulation, they are sufficient in the sense that if returns to capital are high (like in Germany in 1945) the factor accumulation will take place, either from domestic sources or from foreign sources. If

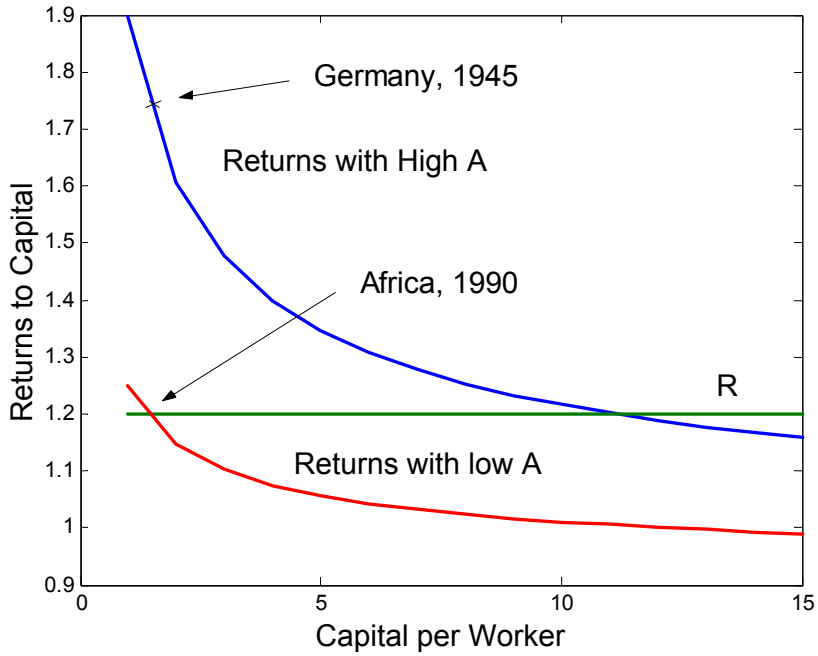

Figure 1: Understanding Returns to capital

returns are low, even if capital accumulation is forcefully imposed, as it has been done in some African countries or in some planned economies, it will not have persistent effects on capital and income per worker.

Figure 2 shows the key determinants of return to capital in various countries (The value are all relative to the US so that a number of 0.90 means 90% of the US level). Observe that for example Uganda has low capital per worker and hence, on the basis on that only, it should have high returns to capital. But Uganda has also low education per worker and low TFP, so that this factors might reduce the return to capital below the level of a country that has a high level of capital per worker (this is why capital does not flow from US to Uganda).

	Capital per worker	Education	TFP
Canada	1.00	0.90	1.03
Italy	1.06	0.65	1.21
France	1.09	0.67	1.13
Japan	1.12	0.80	0.66
Mexico	0.87	0.54	0.93
Chile	0.99	0.66	0.40
Peru	0.94	0.62	0.41
India	0.71	0.45	0.27
Kenya	0.75	0.46	0.17
Uganda	0.36	0.39	0.22

Figure 2: Understanding Returns to capital

Measuring α

One missing from this analysis is the value of α. We'll generally use $\alpha = 1/3$. Why? For this production function, if capital and labor are both paid their marginal products (i.e. if they are paid in proportion to how much they contribute to production), then a fraction α of output is paid to capital and the complementary fraction $1 - \alpha$ is paid to labor. In most countries, the relevant fractions are roughly one-third and two-thirds (recall the income side of the National Income and Product Accounts), hence the choice.

If one considers more general production functions that can display regions of increasing returns (meaning that the return on your capital increases when there is more capital around) then returns to capital becomes even more crucial determinant of growth. Economist Bill Easterly's book the elusive quest for growth gives a nice example on how this can be the case in the case of investment in human capital. Think about the returns from getting an MBA: they are clearly (positively) affected

by the number of other MBAs around you. If I live in a poor country where nobody is getting an MBA, my individual returns are low and thus I will not get it, but since everybody does the same, returns stay low and investment in education never happens.

Growth accounting

So far we have treated total factor productivity as a fixed parameter. This implies that the Solow model (with endogenous or exogenous saving) predicts that once an economy has reached its steady state the growth in per capita income should stop. In the real world we see countries showing sustained growth in per capita income. A sustained growth can be analyzed in the context of the Solow model if one imagines that the total factor productivity parameter A in front of the production function displays growth.

If we take logs of the above expression we have for period t

$$\log(Y_t) = \log(A_t) + \alpha \log(K_t) + (1-\alpha)\left(\log(L_t) + \log(H_t)\right)$$

and for period $t-1$

$$\log(Y_{t-1}) = \log(A_{t-1}) + \alpha \log(K_{t-1}) + (1-\alpha)\left(\log(L_{t-1}) + \log(H_{t-1})\right)$$

subtracting the second expression from the first

$$\begin{aligned}\log(Y_t) - \log(Y_{t-1}) &= \log(A_t) - \log(A_{t-1}) + \\ &\quad \alpha\left(\log(K_t) - \log(K_{t-1})\right) + \\ &\quad (1-\alpha)\log(L_t) - \log(L_{t-1})) + \\ &\quad (1-\alpha)\left(\log(H_t) - \log(H_{t-1})\right)\end{aligned}$$

now remember that the growth rate of a variable

$$g_y = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{Y_t}{Y_{t-1}} - 1$$

which implies

$$1 + g_Y = \frac{Y_t}{Y_{t-1}} \tag{2}$$

remember that

$$\log(1+x) \simeq x \tag{3}$$

so that taking logs on both sides of 2 and using 3 we get

$$\log(\frac{Y_t}{Y_{t-1}}) = \log(1+g_Y) \simeq g_Y$$

that implies

$$g_Y \simeq \log(Y_t) - \log(Y_{t-1})$$

and thus we can decompose growth in income as follows

$$g_Y = g_A + \alpha g_K + (1-\alpha)(g_L + g_H) \tag{4}$$

that also implies

$$g_A = g_Y - \alpha g_k - (1-\alpha)(g_L + g_H)$$

The last equation implies that give an estimate of α we can measure g_A that is TFP growth. Total Factor Productivity growth captures the growth in output (g_y) that is not explained by the growth in the factors of production (g_k, g_L or g_H). This type of decomposition is called *growth accounting*. Notice that growth in total factor productivity also explains how can a country continue to grow once it has reached its steady state level. It is interesting to look at various region in the world that have experienced different growth patterns and identify the sources of their growth:

Sources of growth in 5 World regions (1960-1994)

	Output	Capital	Labor (Adj. for education)	TFP
	g_y	αg_k	$(1-\alpha)(g_L + g_H)$	g_A
East Asia	6.97	2.88	$1.63 + 0.55$	1.69
South Asia	5.33	1.92	$1.30 + 0.34$	1.36
Africa	3.09	1.94	$1.21 + 0.22$	-0.62
Latin America	3.64	2.72	$0.98 + 0.22$	-0.29
Industrialized	3.42	2.40	$0.35 + 0.17$	0.41

From the previous table notice that the high level of growth of the East Asian tigers have been sustained by high level of capital stock growth (through investment), by high level of labor growth (through population growth, increase in the participation rate and increases in education) and high increases in total factor productivity. From our previous discussion on returns to capital we can conclude that the significant growth rates of TFP have been the prerequisites for the high rate of accumulation of the other factors. The table also shows that the key difference between Asia v/s Africa and Latin America has really been the growth rate in TFP (significant in one case and negative in the other).

Another interesting example of the importance of TFP is the case of Italy. Figure 3 plots Real GDP, Total Labor Input, Capital and TFP for Italy from 1960 to 2010. Note two things: first during the so-called Italian Miracle (1960-1980) GDP grew at a very rapid pace, despite a falling labor input: what has fueled GDP growth has been rapid TFP growth. Second, the period 1990-2010 (which could be called the "Italian Disaster") GDP stagnated despite growth in labor input and in capital; once again the cause has been dismal growth in TFP. Overall the picture suggests that a key

driver of macroeconomic performance is not so much the measurable accumulation of capital nor the increase in labor input, but rather TFP, i.e. the ability of producing more output with the existing measurable factors.

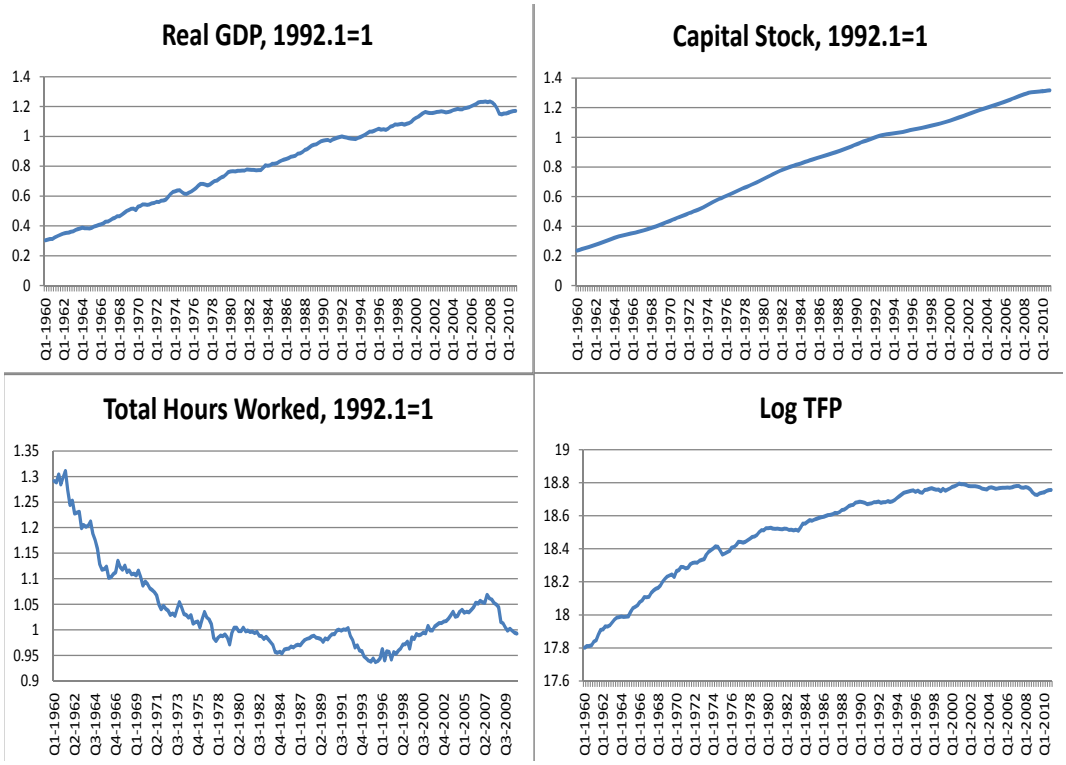

Figure 3: The Italian Miracle and the Italian Disaster

Note that equation 4 tells us how to account for the growth in output. How would we account for the growth rate of the more conventional measure of macroeconomic performance, GDP per worker? or of GDP per capita? To account for the growth rate of GDP per worker simply add and subtract g_Y from both sides of 4 and obtain

$$g_Y - g_L = g_A + \alpha(g_K - g_L) + (1-\alpha)g_H \tag{5}$$

Now using the properties of growth rates derived above one gets

$$g_{Y/L} = g_A + \alpha g_{K/L} + (1-\alpha)g_H$$

regarding per capita income let POP stand for population. Then output per capita is

$$Y/POP \;=\; (Y/L)(L/POP).$$

In growth rates, we simply add an extra term to 4:

$$\begin{aligned} g_{Y/POP} &= g_{Y/L} + g_{L/POP} \\ &= g_A + \alpha g_{K/L} + (1-\alpha)g_H + g_{L/POP}. \end{aligned}$$

The ratio L/POP is the employment rate: the fraction of the population that is working.

The importance of TFP growth

In the Krugman Foreign Affair article (which is a required reading and posted on the class page) it is stated that growth driven by factor accumulation is bound to be limited by the decreasing return to capital. But if the growth is driven by TFP this reasoning does not apply. From equation 1 above we can see that increases in A raise the MPK while increase in K reduce the MPK. One way in which we can distinguish what type of growth are countries experiencing is to look at prices instead of quantities. If the growth is indeed driven only by factor accumulation we should observe a decline in the returns to capital, if the growth is also fueled by TFP we should not observe very large decline in returns to capital. A recent paper (Chang-Tai Hsieh, What Explains the Industrial Revolution in East Asia? Evidence from the Factor Markets. American Economic Review, June 2002) has attempted to measure return to capital by looking at interest rates in Singapore and Taiwan. He found that looking at interest rates (instead of looking simply at quantities) tends to reduce the role of factor accumulation in favor of a larger role of TFP, as in countries that experience fast growth like Taiwan and Singapore we do not observe large reduction in returns to capital.

Revised Estimate of TFP growth in Asian Countries

	Growth Rates of:			
	Output per worker	Returns to Capital	TFP (Old Est.)	TFP (New Est.
Singapore	4.30	-0.07	2.60	3.79
Taiwan	4.20	1.64	-0.22	2.16

Many other recent studies stress the role of TFP over factor accumulation per-se

- Bill Easterly has shown that about 2/3 of the variation of growth in a large cross section of countries (from 1960 to 1994) is explained by TFP growth. He also shows that the growth rate in GDP is not very persistent across decades and the growth of TFP share the same characteristic while factor accumulation is usually very persistent across decades.

- Many authors have shown that developing countries face much larger output fluctuations than developed countries and that these fluctuations are driven by TFP fluctuations. So TFP is not only important for understanding long run growth but also shorter run fluctuations.

The key point is thus that TFP is a key variable to look at when we are trying to understand the long run growth performance of countries.

Concepts you should know

1. Returns to capital
2. Growth accounting
3. Role of TFP in determining economic growth